



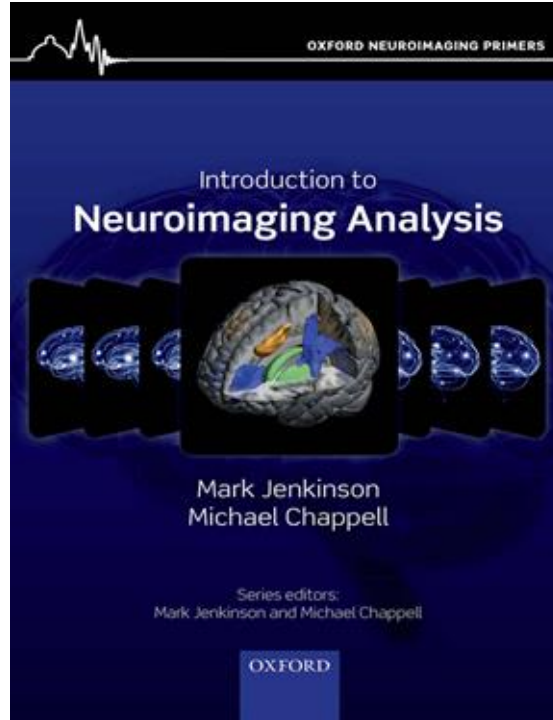
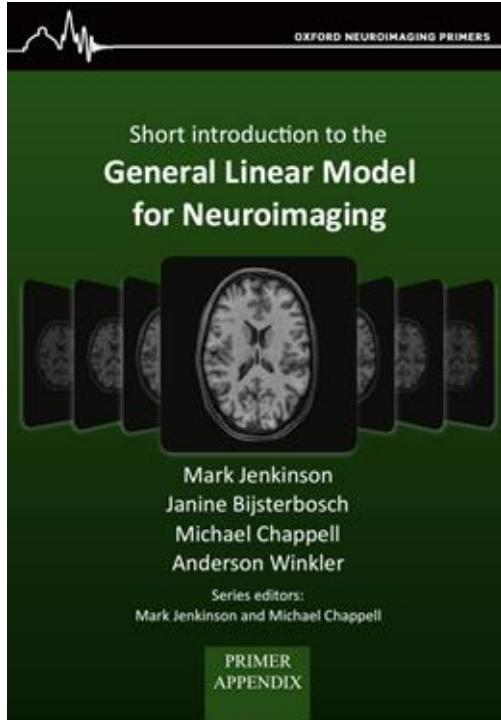
GENERAL LINEAR MODEL FOR FMRI ANALYSIS

Turku PET Centre Brain Imaging Course 2026

Maya Rassouli, Turku PET Centre

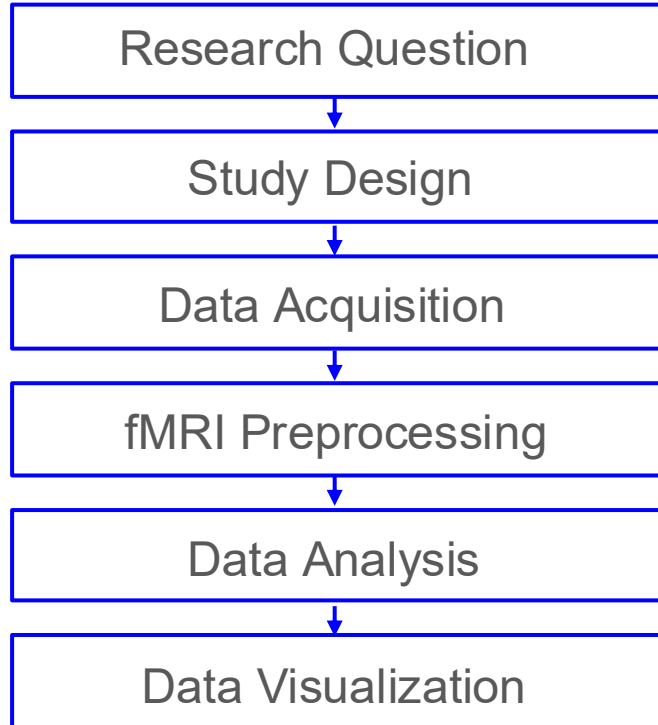


Resources



[Principles of fMRI Part 1 Module 15: The General Linear Model - Intro](#)

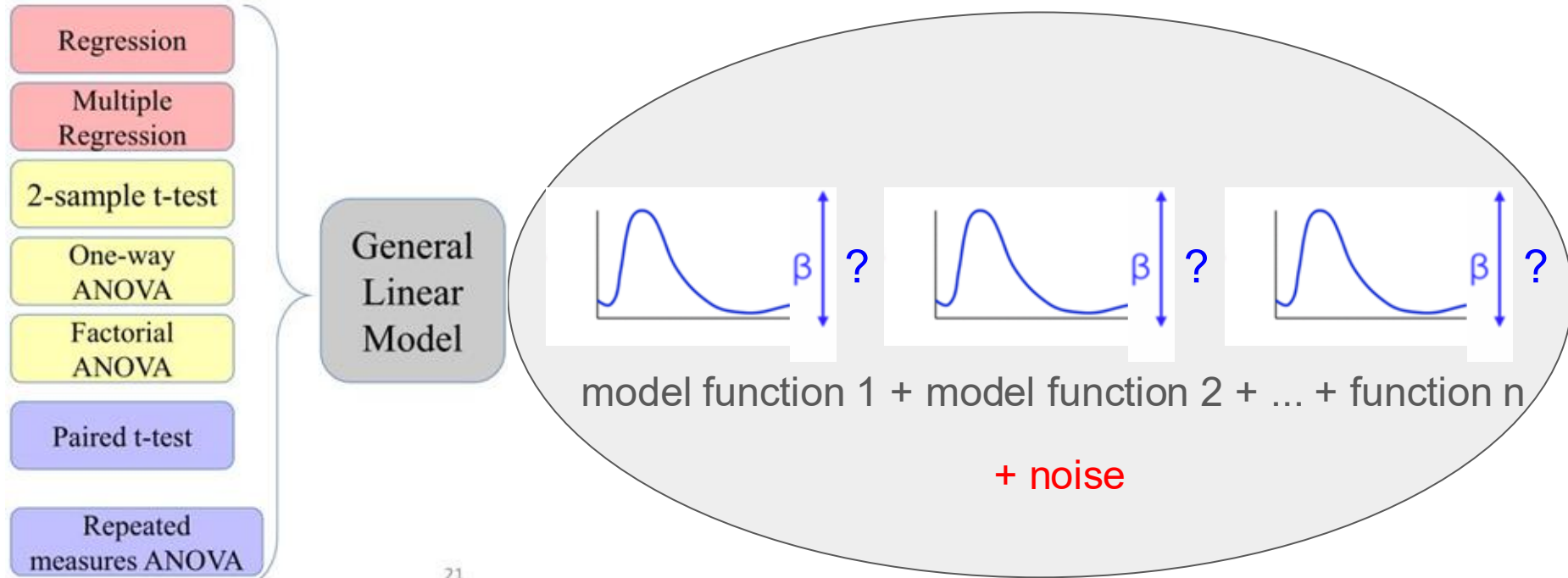
[Principles of fMRI Part 1, Module 16: GLM applied to fMRI](#)



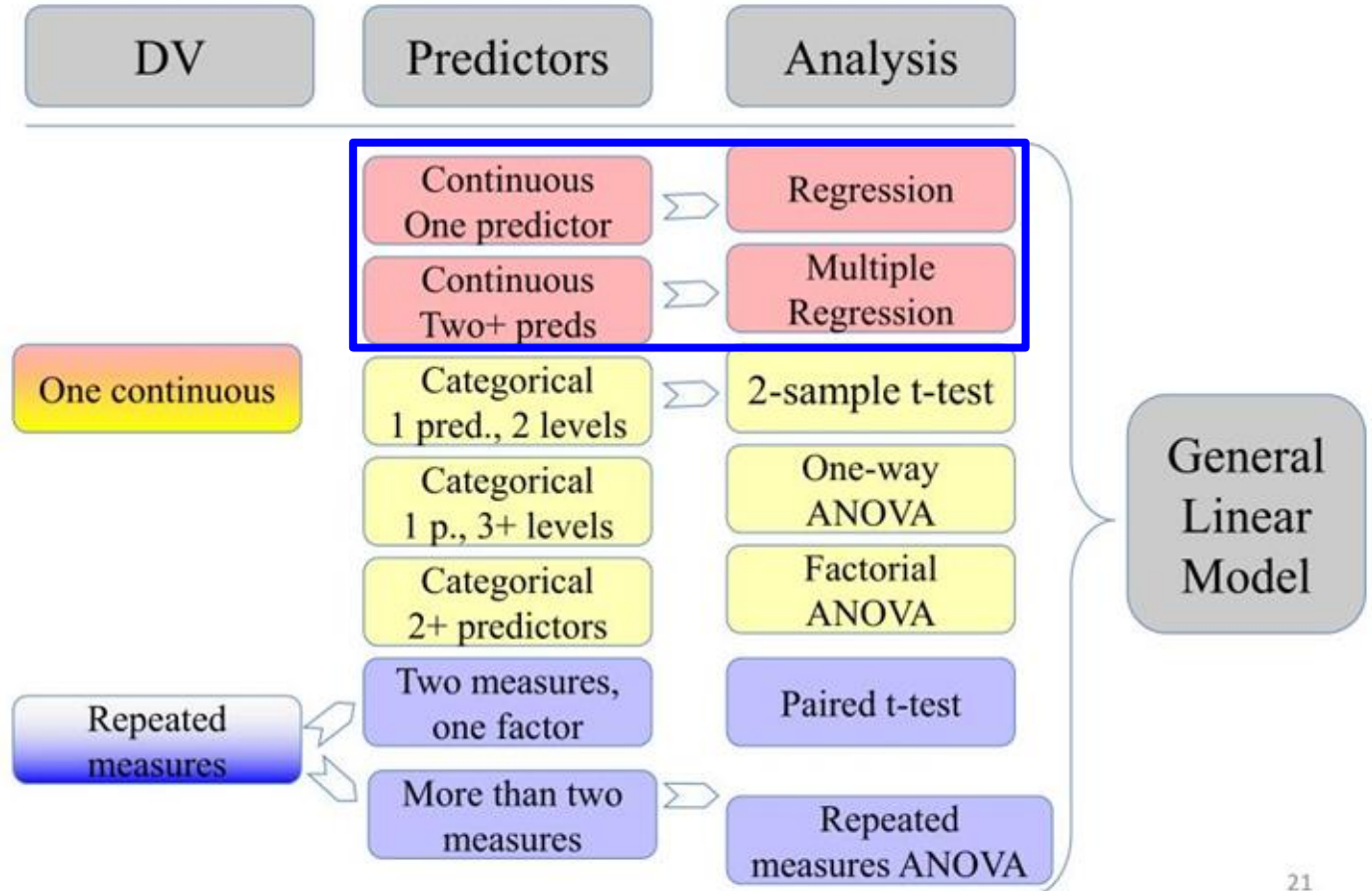
Outline

1. General Linear Model (GLM) family
2. GLM for fMRI Analysis
 - a. Simple Regression
 - b. Multiple Regression
 - c. Confounds
 - d. Limitations
3. Case Study & Quiz

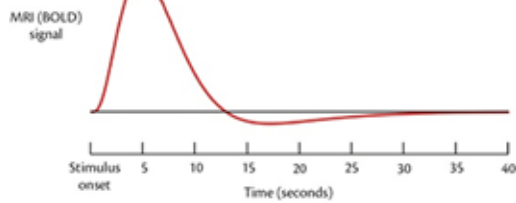
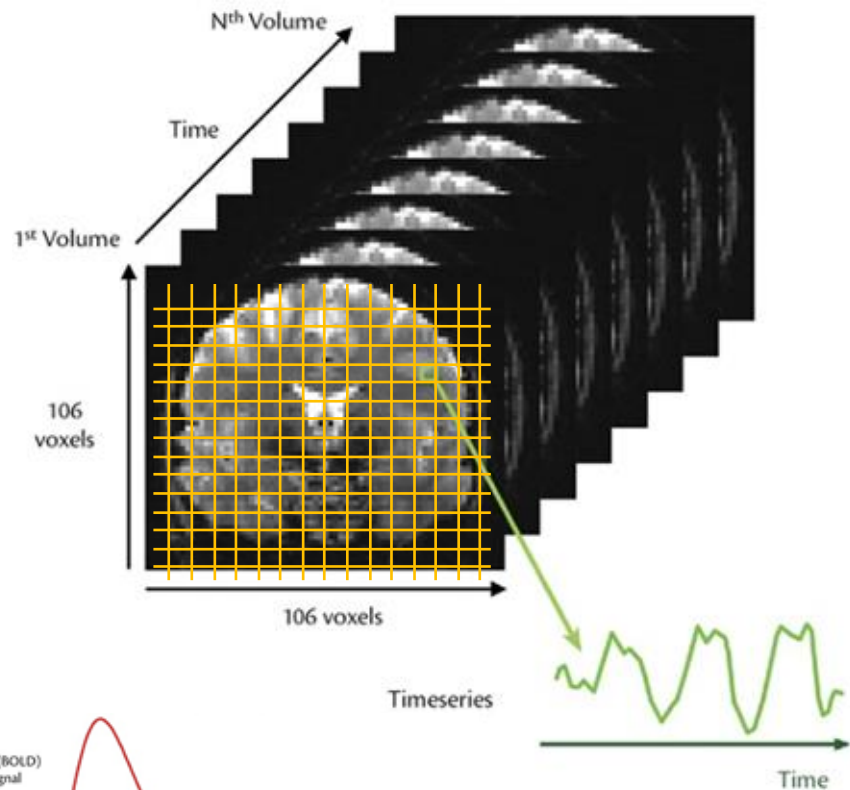
GLM family



GLM family



Reminder

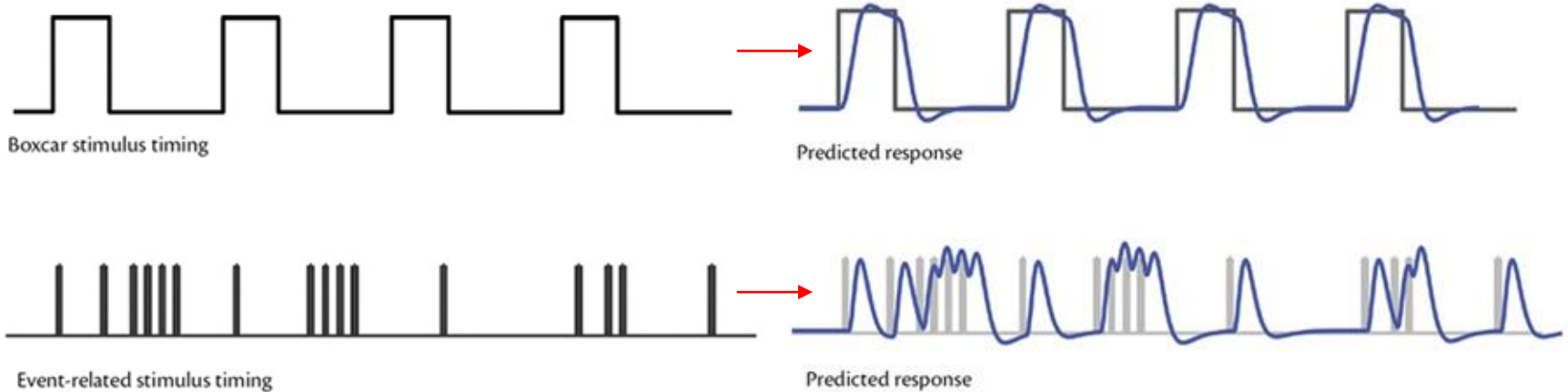
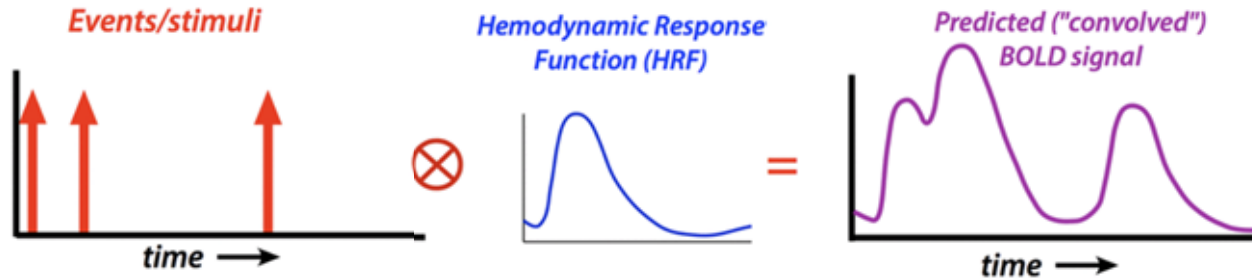


GLM for fMRI Analysis

GLM

- looks for the predicted responses in the measured fMRI signal.

Predicted response model



GLM for fMRI Analysis

GLM

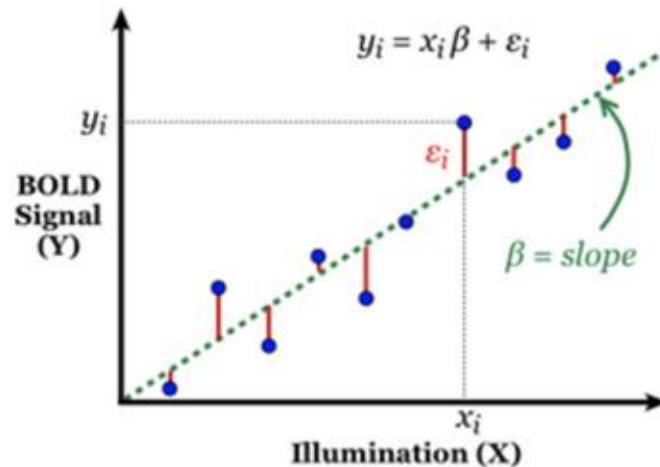
- looks for the predicted responses in the measured fMRI signal.
- helps determine where there is significant relationship between the measured signal and the predicted response

Simple Regression

$$Y = X\beta + \varepsilon$$

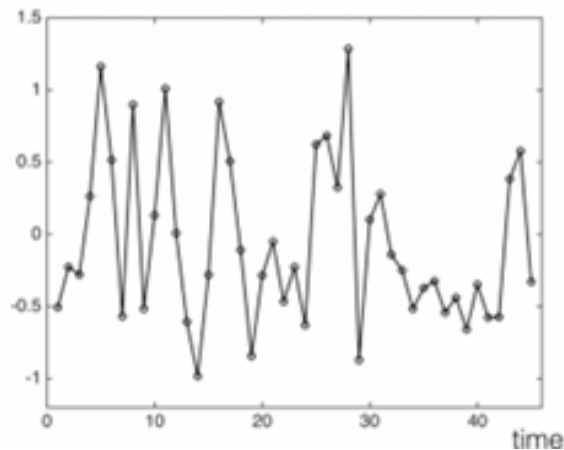
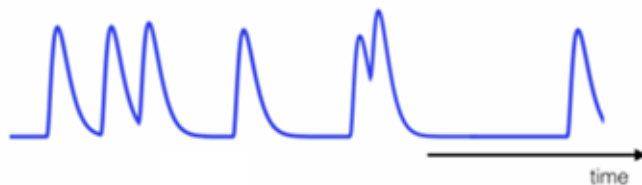
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} [\beta] + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

- Y = BOLD timeseries
- X = regressor
- β = scaling parameter
- ε = residual error / residuals



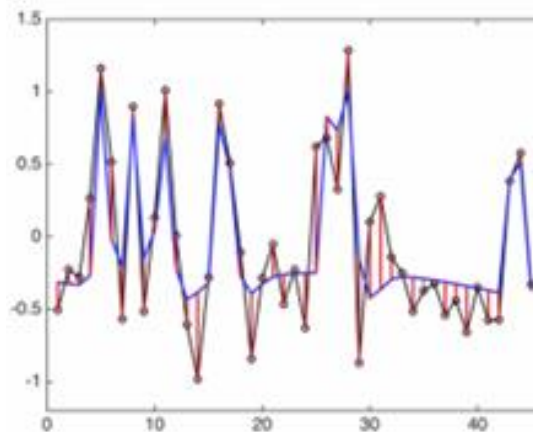
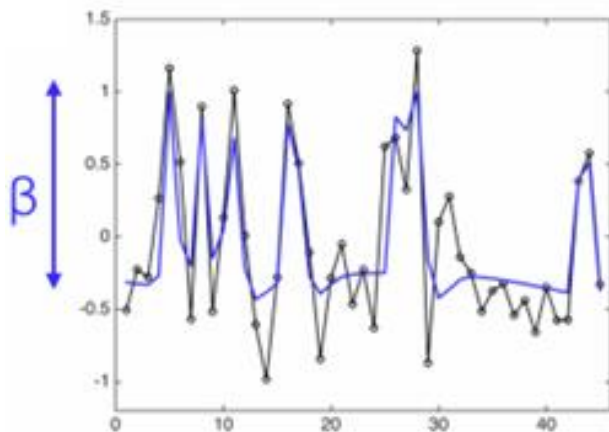
Simple Regression

predicted
response



BOLD
timeseries

GLM
fitting

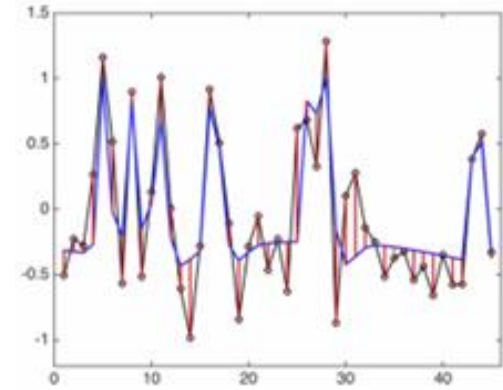


$$Y = X \beta + \epsilon$$

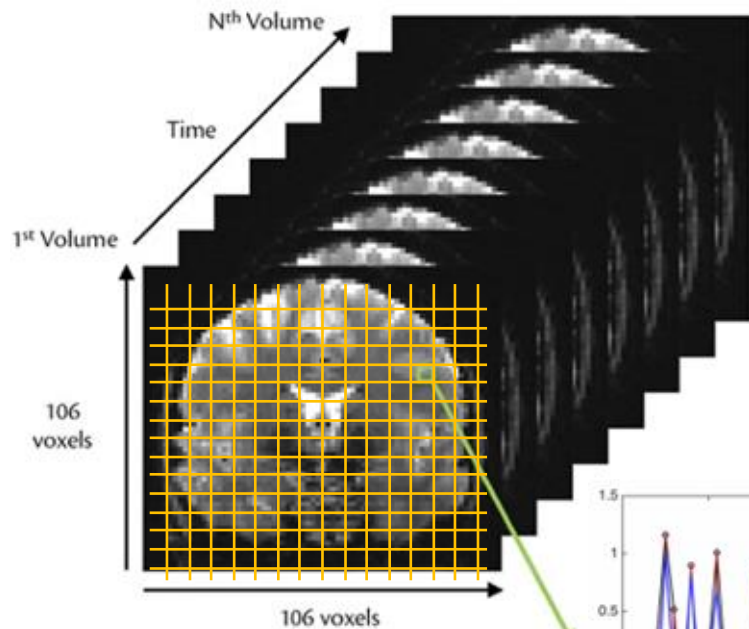
“Best fit”

Difference between the residuals are minimised

→ Minimising the residuals / Finding the least squared error

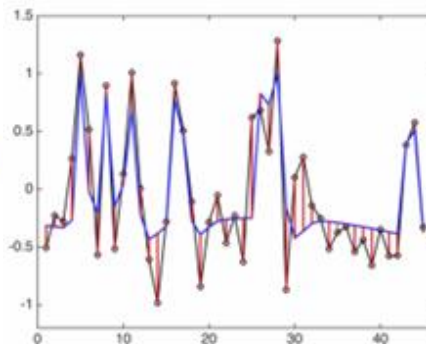


Voxelwise + Mass Univariate Analysis



$$\begin{aligned}
 y_1 &= x_1\beta + \varepsilon_1 \\
 y_2 &= x_2\beta + \varepsilon_2 \\
 y_3 &= x_3\beta + \varepsilon_3 \\
 &\vdots \\
 y_n &= x_n\beta + \varepsilon_n
 \end{aligned}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} [\beta] + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$



Multiple Regression

$$\begin{array}{c} \text{BOLD} \\ \text{Timeseries} \end{array} \quad \begin{array}{c} \text{Design Matrix} \\ \text{Regressors of Interest} \end{array} \quad \begin{array}{c} \text{Amplitude} \\ \text{parameter} \\ \text{(unknown)} \end{array} \quad \begin{array}{c} \text{Error} \\ \text{Term} \end{array}$$

$$\begin{array}{c} \text{time} \downarrow \\ \left[\begin{array}{c} Y_1 \\ \vdots \\ Y_n \end{array} \right] = \left[\begin{array}{ccc} X_{1,1} & X_{1,2} & X_{1,3} \\ \vdots & \vdots & \vdots \\ X_{n,1} & X_{n,2} & X_{n,3} \end{array} \right] \times \left[\begin{array}{c} \beta_1 \\ \vdots \\ \beta_p \end{array} \right] + \left[\begin{array}{c} \epsilon_1 \\ \vdots \\ \epsilon_n \end{array} \right]
 \end{array}$$

Multiple Regression

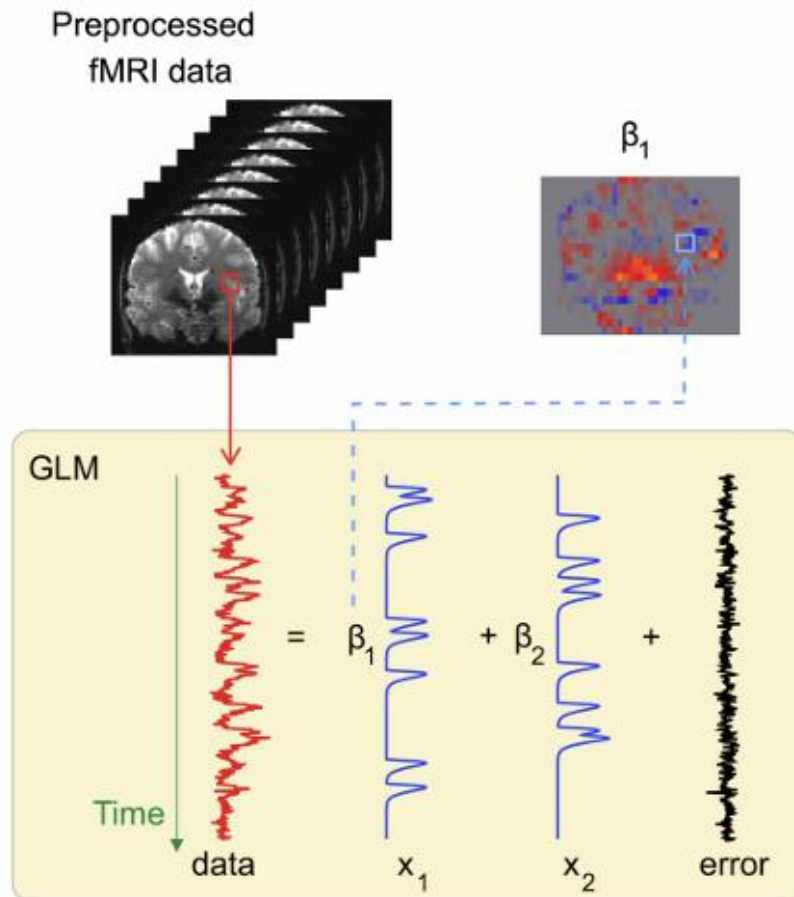
$$Y = X\beta + \epsilon$$

time
↓

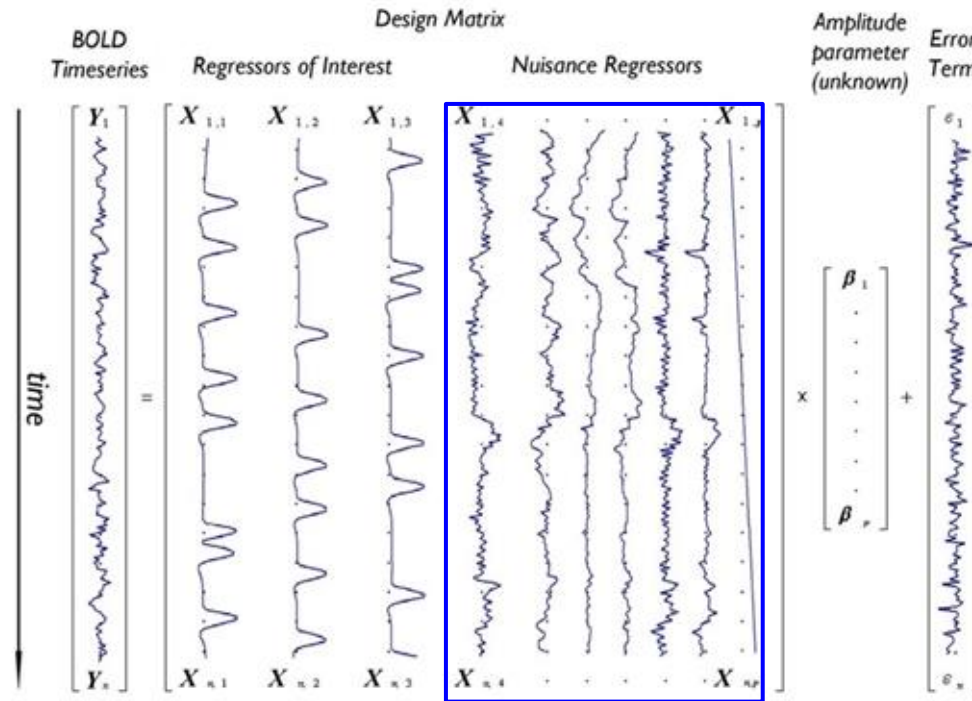
$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1p} \\ 1 & X_{21} & \cdots & X_{2p} \\ \vdots & \vdots & & \vdots \\ 1 & X_{np} & \cdots & X_{np} \end{bmatrix} \times \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

- Y = BOLD timeseries
- X = design matrix of all the regressors
each regressor = one column
- β = vector of all the individual scaling parameters
- ϵ = residual error / residuals

Results



Confounds / Nuisance Regressors

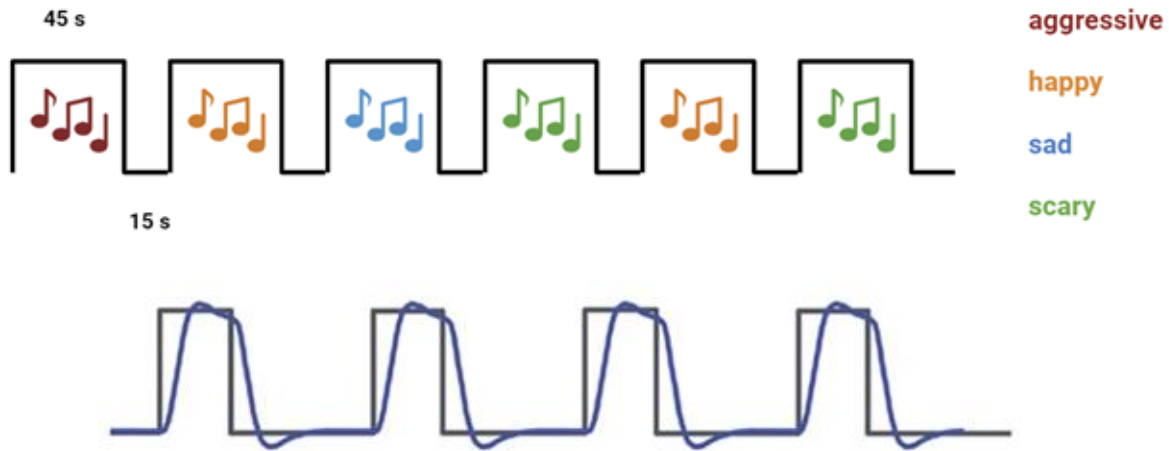
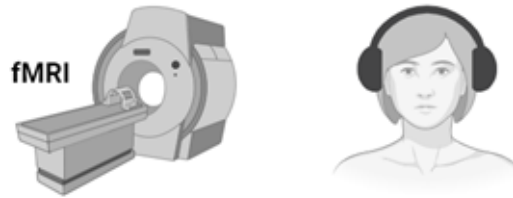


Depiction of the General Linear Model (GLM) for a voxel with time-series Y predicted by a design matrix X including 10 effects (three regressors of interest – e.g., tasks A,B,C – and seven nuisance regressors – e.g., six motion parameters and one linear drift). Calculated weighting factors ($\beta_1 - \beta_{10}$) corresponding to each regressor are placed in amplitude vector β while column vector ϵ contains calculated error terms (ϵ_i) for the model corresponding to each time point i . (From Monti, 2011, under CC BY license)

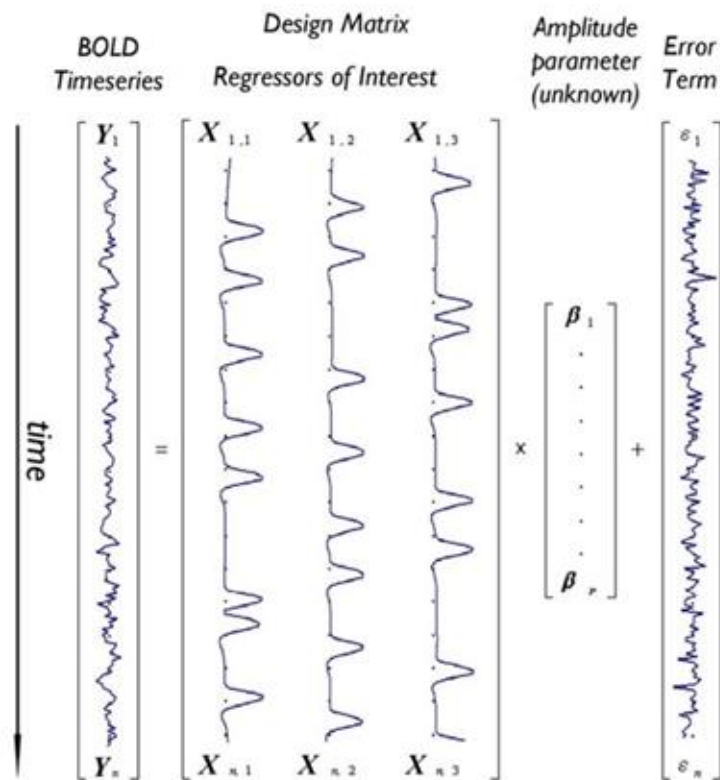
Limitations

- The model is the same for each voxel (mass univariate), but the haemodynamic response might differ across the brain regions.
- The model assumes that noise varies with a normal distribution in each voxel in each time point, but e.g. noise differs greatly close to large arteries.
- The model assumes independent statistical test for each voxel, but voxels in the same vicinity have similar properties.

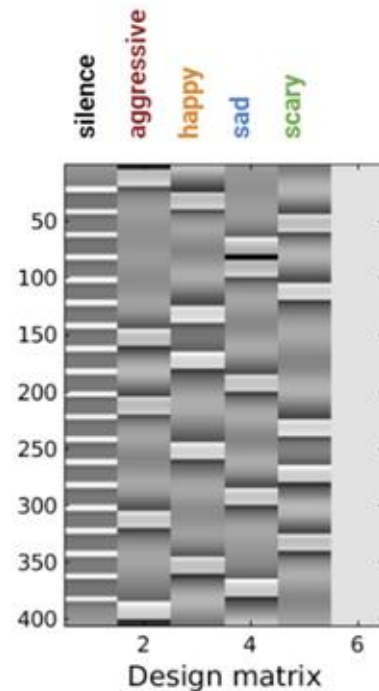
Case Study



Case Study



GLM Design Matrix



Quiz 1

How do you get the predicted response model ?

By convolving the known stimuli design with the haemodynamic response function

Quiz 2

How does one fit the best GLModel possible ?

- 1) Minimising the residuals
- 2) Including relevant confounds in GLM

Quiz 3

Will including an additional regressor in the GLM affect the parameter estimates ?

Yes

Take Home Messages

fMRI GLM analysis

- determines where there is a significant relationship between the measured BOLD signal and the predicted response
- provides a set of whole brain maps of beta values (one for each regressor)



Thank you for your attention !

Turku PET Centre Brain Imaging Course 2026

Maya Rassouli, Turku PET Centre

