



Turku PET CENTRE

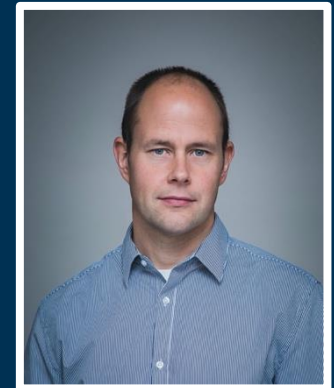
PSYK8965 Turku PET Centre Brain Imaging Course

Basics of biological modelling with PET data

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Turku PET Centre, University of Turku

31st August 2026





About this lecture (what you should learn)

Contents of the talk

- PET imaging
- Image segmentation
- PET modelling
- Interpretation

Key terminology

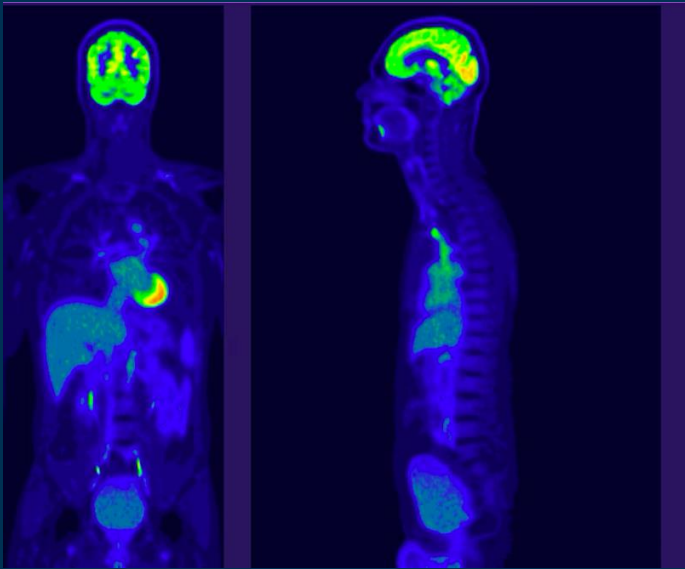
- Dynamic PET
- TAC
- Compartment model
- Patlak plot
- Logan plot
- Parametric imaging

Why do we need modelling?

- PET images can contain millions of pixels (or voxels or data points)
 - Big data
- PET images are 3D (static) or 4D (dynamic)
 - Hard to visualise
- Analysis is complex
 - Automation helps (a little)
- Conclusions are difficult to make
 - Modelling helps (a lot)
- Modelling is needed to extract important information

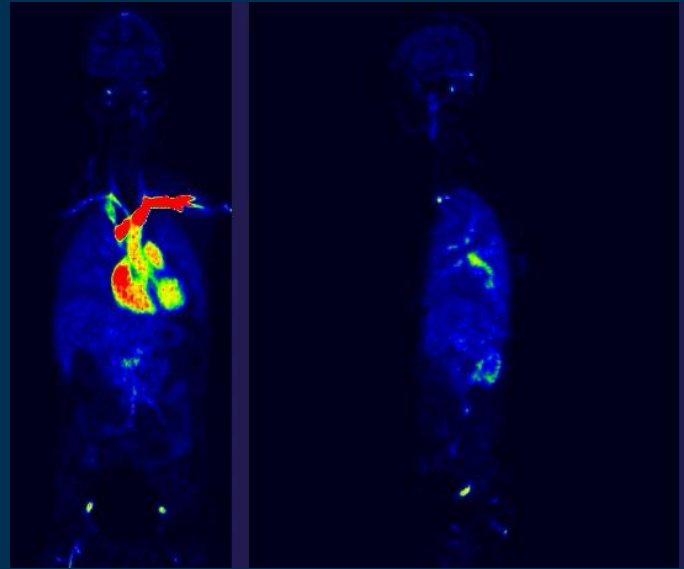
Static and dynamic imaging

Static PET imaging



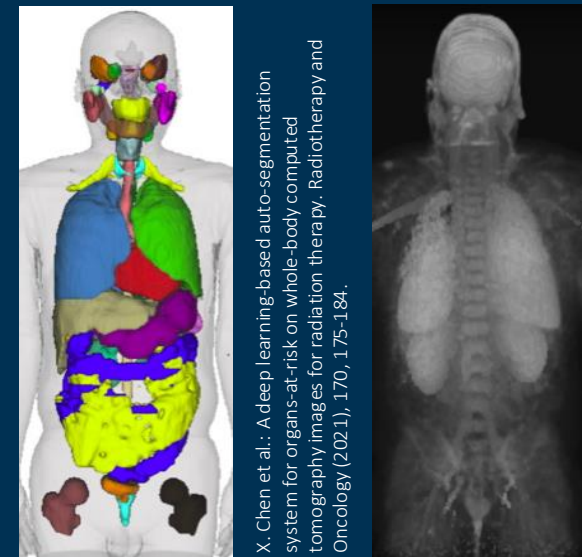
40 minutes

Dynamic PET imaging



40 minutes, 13 frames

Image analysis



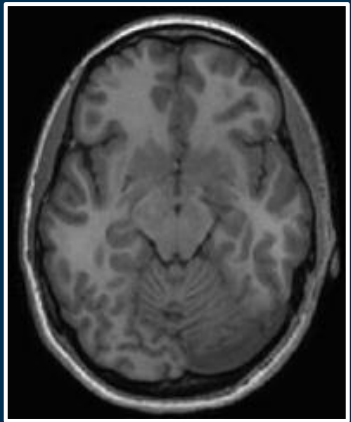
X. Chen et al.: A deep learning-based auto-segmentation system for organs-at-risk on whole-body computed tomography images for radiation therapy. Radiotherapy and Oncology (2021), 170, 175-184.

Segmentation, modelling

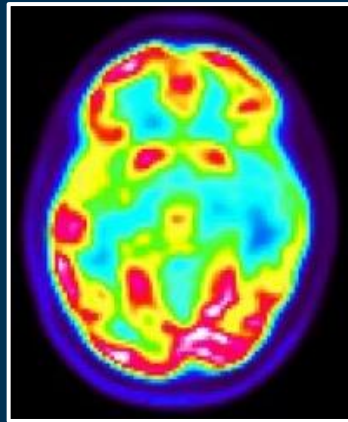
These example images were provided by Prof. Kirsi Virtanen.

Why PET imaging?

To understand how
organism functions



MRI



PET

Clinical motivation

- Locate diseases, e.g. cancer
- Find malfunctions, e.g. ischemia

Understanding physiology

- Neuroimaging, e.g. emotions
- Metabolic imaging, e.g. brown fat

Research

- Drug development, e.g. distribution



PET imaging flow

Radioactive tracer

- Various different tracer for different purposes, e.g. radioactive water for blood flow, glucose (FDG) for metabolism
- Produced with cyclotron and radiochemistry lab
- Injection (or inhale)

PET imaging

- Duration from a couple of minutes to tens of minutes
- Patient lays still
- Typically with CT or MRI

Image analysis

- Segmentation
- Visual inspection
- Modelling
- Results

Image segmentation

Manual segmentation

- 2D tools (e.g. paintbrush)
- Tedious & subjective (low repetitive)
- Human control

Semi-automatic segmentation

- 2D or 3D tools (e.g. region grow algorithm)
- Less tedious & subjective
- Some human control

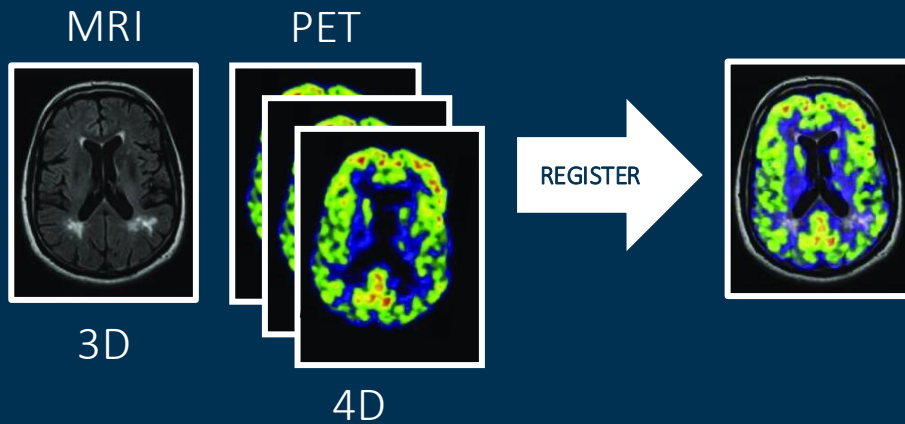
Automatic segmentation

- 2D, 3D or 4D methods (e.g. deep learning)
- Fast & objective (repetitive)
- No human control

PET-MRI image segmentation in neuroimaging

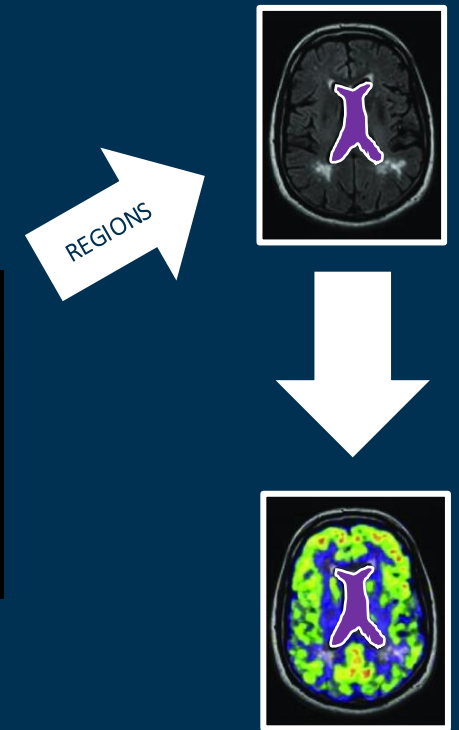
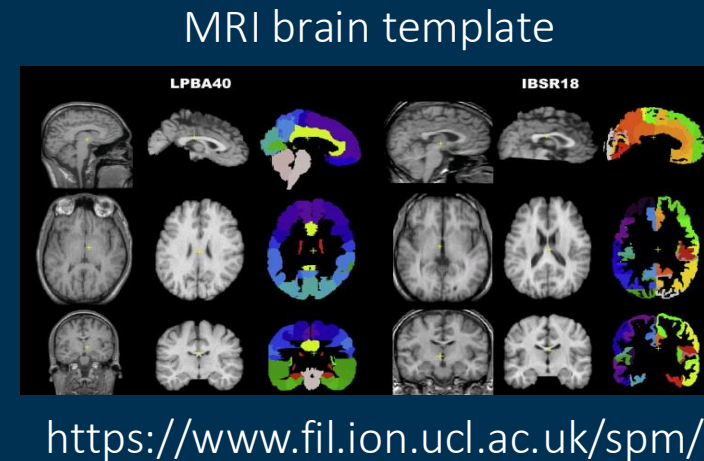
Automatic PET-MRI registration

- Dynamic PET and MRI images are registered

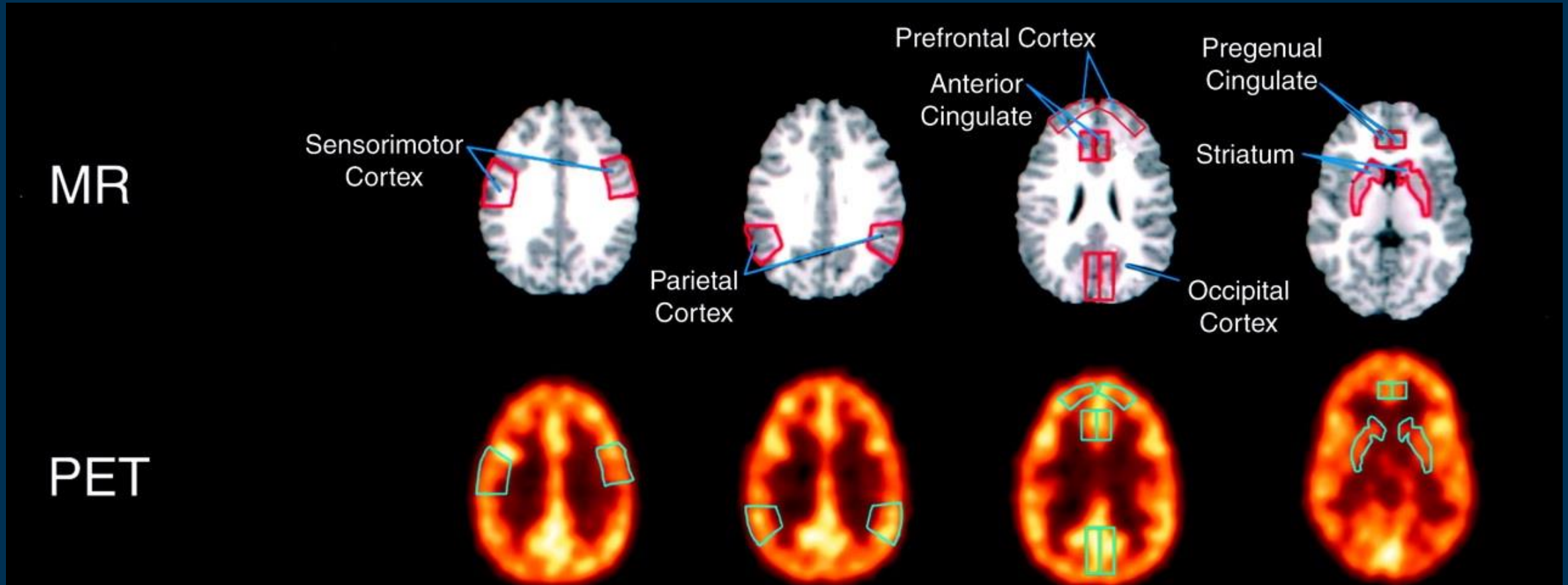


Automatic MRI segmentation

- Manually segmented MRI templates



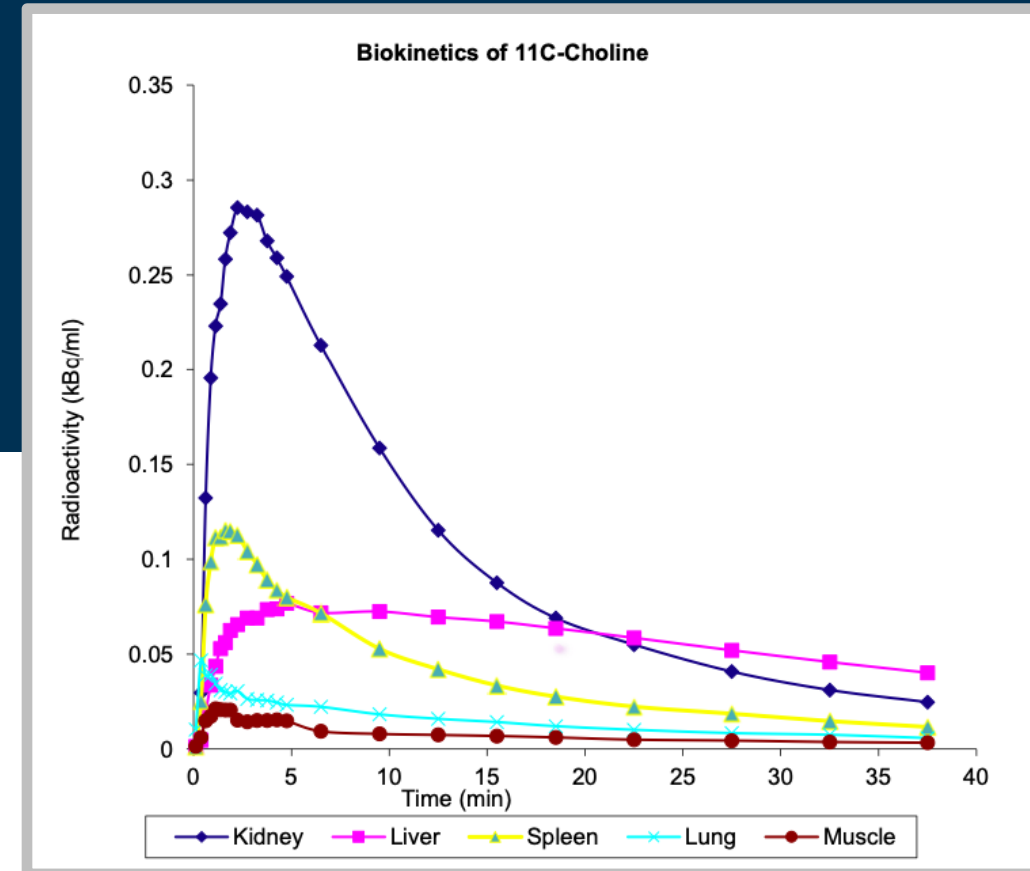
PET-MRI image segmentation in neuroimaging



<https://www.fil.ion.ucl.ac.uk/spm/>

Time Activity Curve (TAC)

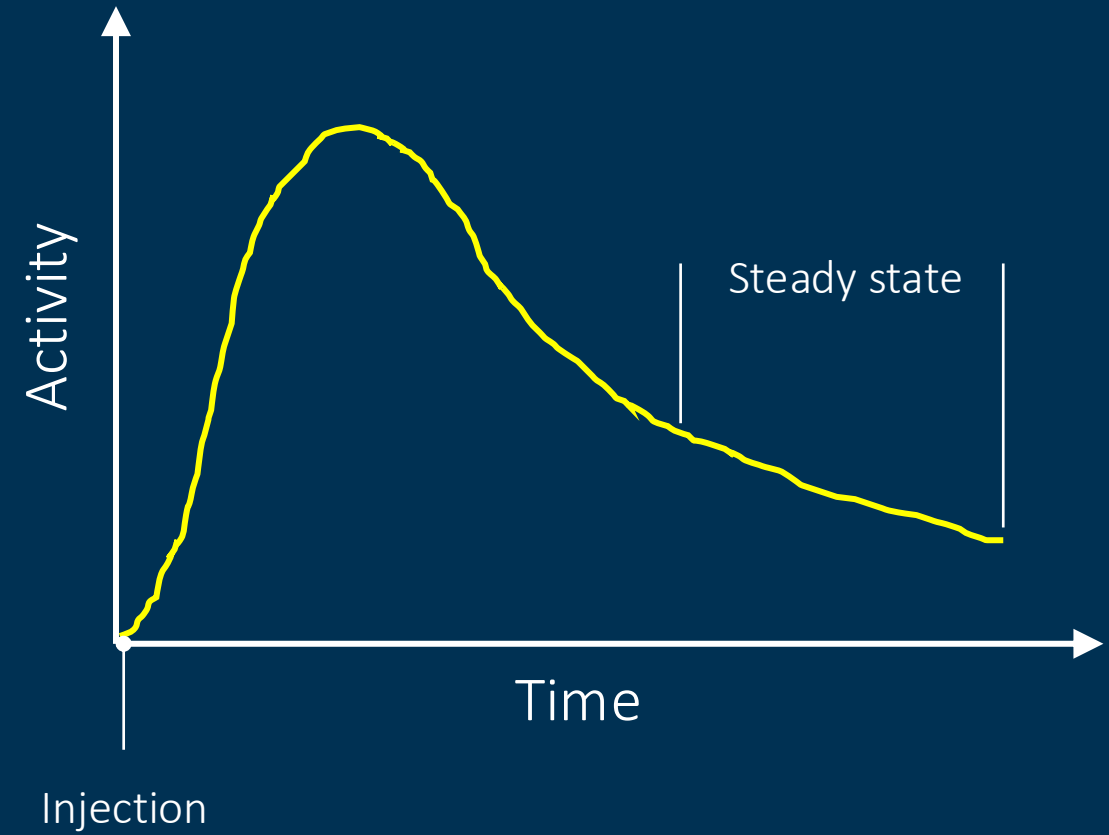
- Dynamic PET images form a 3D video over time
- Activity in a PET image for the same region in different time is called Time Activity Curve (TAC)
- The region can be e.g.
 - Segmented tissue or organ
 - A single voxel
- TAC illustrates how the tracer behaves over time
- TACs in different regions have different behaviour



T. Tolvanen: Studies on dosimetry of positron emitting radiopharmaceuticals. Doctoral dissertation, 2023.

Modelling of tracer kinetics

- Tracer injection and steady state
- Injection time(s) and imaging time
 - Depending on tracer, imaging can be done several minutes after the injection
- Delay: tracer reaches some tissues earlier than others
- Tracer behaviour
 - Reversible uptake
 - Irreversible uptake





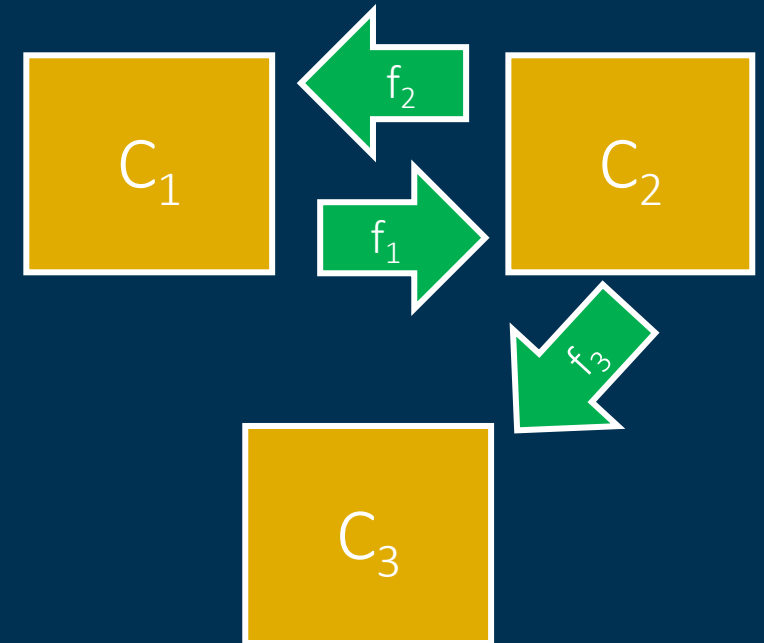
Idea of PET modelling

Tissues are considered as separate compartments assuming

- Uniform concentration in each compartment
- Tracer flow from a compartment is relative to its concentration
- Tracer does not change physiology
- Labelling the tracer molecule with a radionuclide does not alter its properties

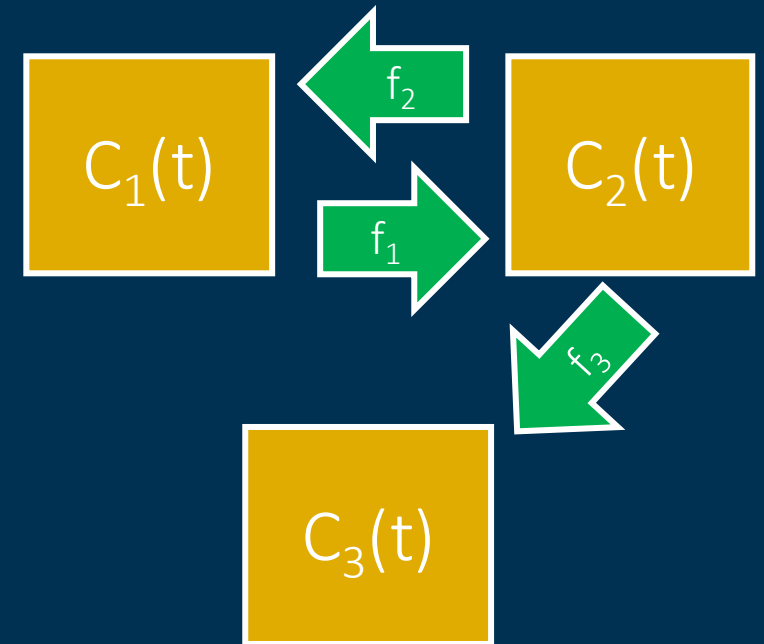
Compartment models

- Compartment is a uniform object with measurable matter (=tracer concentration)
 - Marked typically as a box (C_1 , C_2 , C_3)
- Flow is a constant that determines movement of the matter
 - Marked typically as an arrow (f_1 , f_2 , f_3)
- Idea of compartment model is to model the flow of the matter in time in each C_i

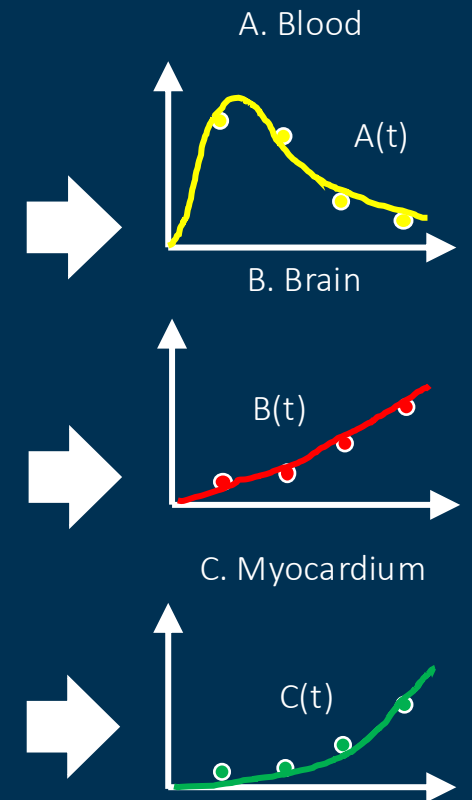
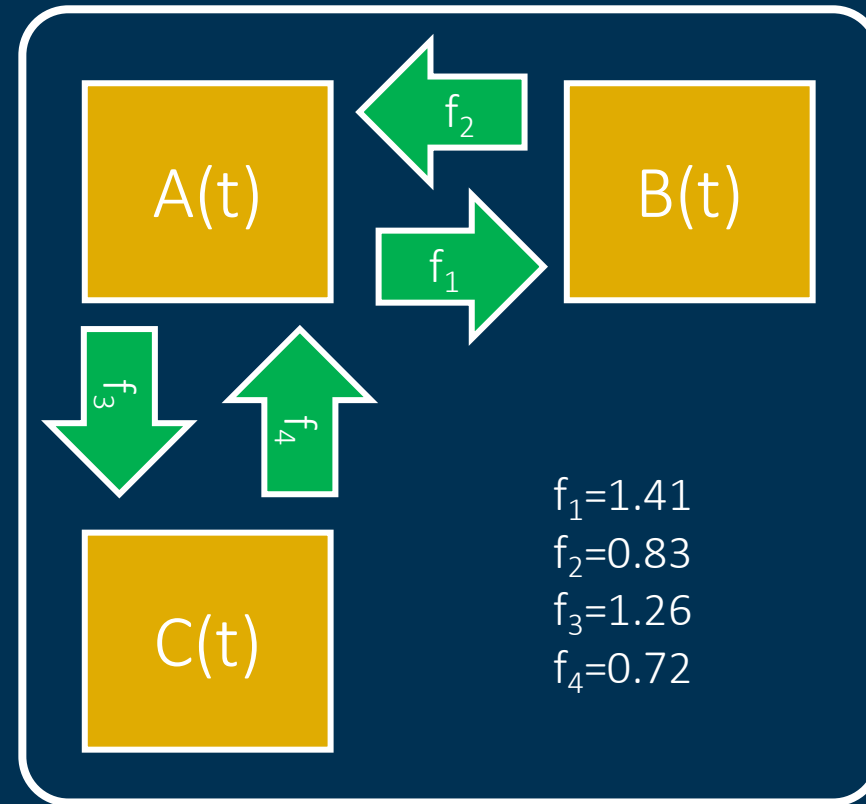
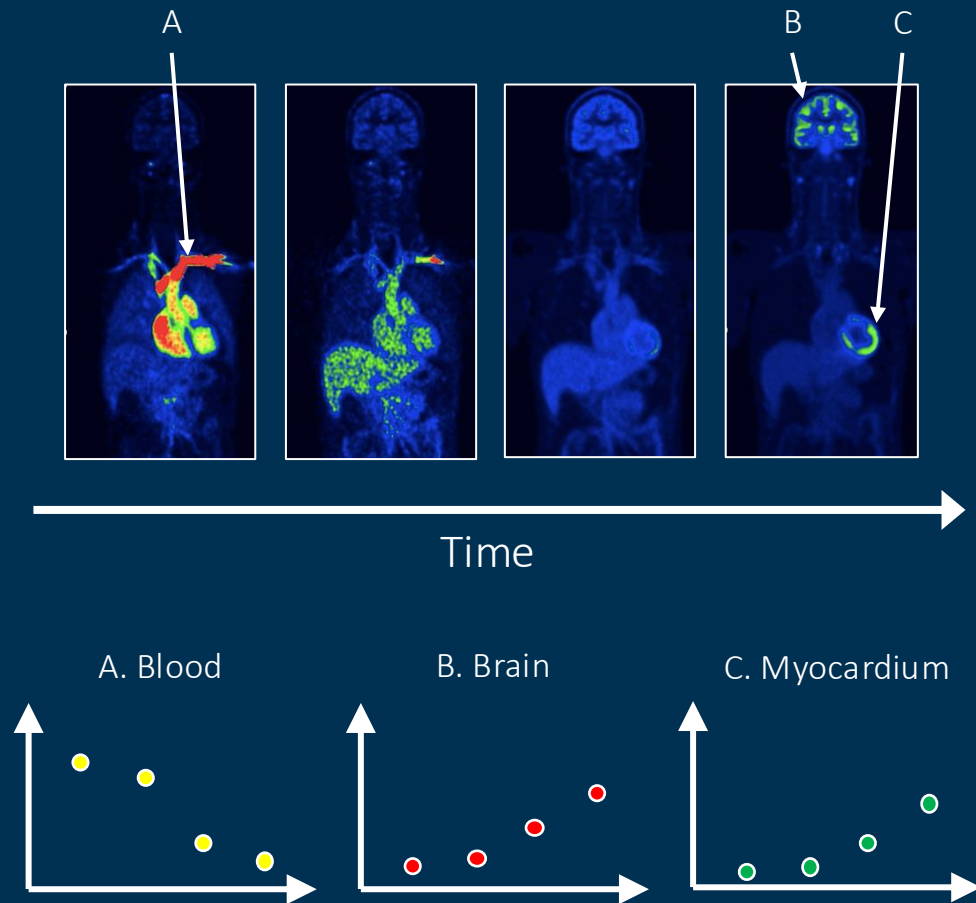


Compartment models, dynamic modelling

- Concentration in a compartment is a function of time [denoted $C(t)$], and flow is constant [f]
- From a PET image we can define values $C_i(t)$ for each measured timepoint
- We are interested in finding the flow constants f_i
- E.g. change of $C_1(t)$ at time t is $C_2(t) f_2 - C_1(t) f_1$ and change of $C_3(t)$ is $C_2(t) f_3$



Compartment models, example



Compartment models, formulas

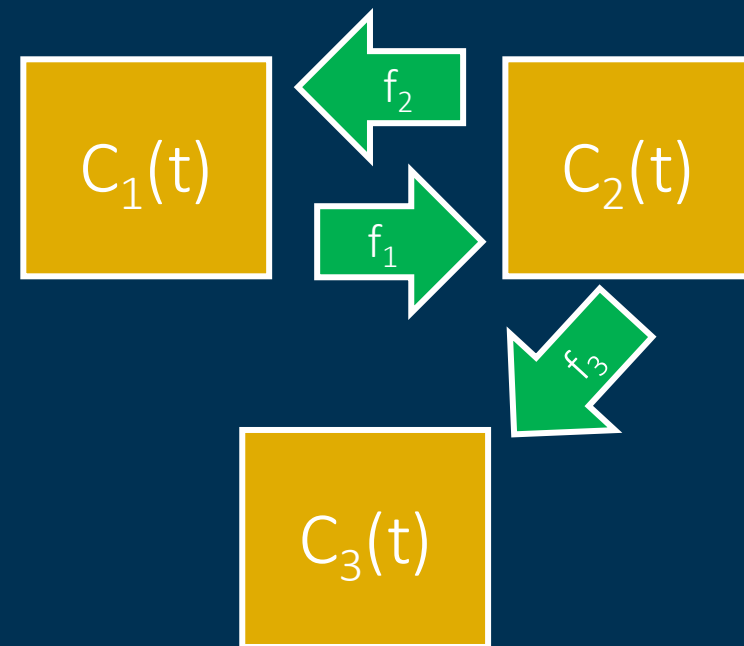
- In mathematical notion the example model becomes a differential equation:

$$\frac{\partial C_1(t)}{\partial t} = C_2(t)f_2 - C_1(t)f_1$$

$$\frac{\partial C_2(t)}{\partial t} = C_1(t)f_1 - C_2(t)f_2 - C_3(t)f_3$$

$$\frac{\partial C_3(t)}{\partial t} = C_2(t)f_2$$

- Fitting the measured $C_i(t)$ values to this model gives the flow constants f_i



Compartment models in PET imaging



Typical models in PET have 2 or 3 compartments

- One compartment is blood
- Other compartments for tissues or organs

Thus 1 tissue compartment model

=2 compartment model

and 2 tissue compartment model

=3 compartment model,

Blood in compartment model is called input function.

It can be measured or estimated from image

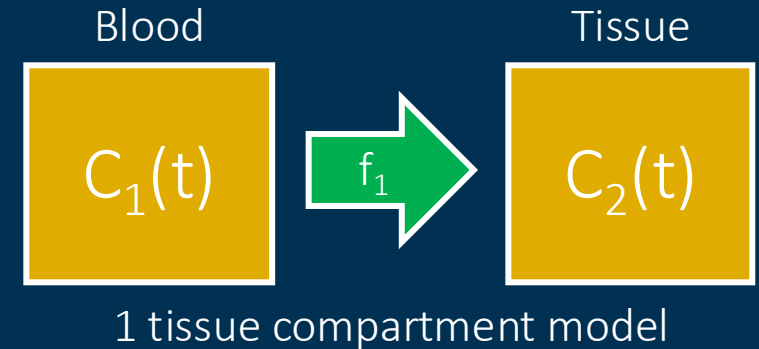
Patlak plot for irreversible uptake, 1TCM

Patlak plot is based on the observation that solution of simple irreversible model satisfies

$$\frac{C_2(t)}{C_1(t)} = A \frac{\int_0^t C_1(s) ds}{C_1(t)} + B$$

In other words, plotting $\frac{C_2(t)}{C_1(t)}$ against $\frac{\int_0^t C_1(s) ds}{C_1(t)}$ gives a line

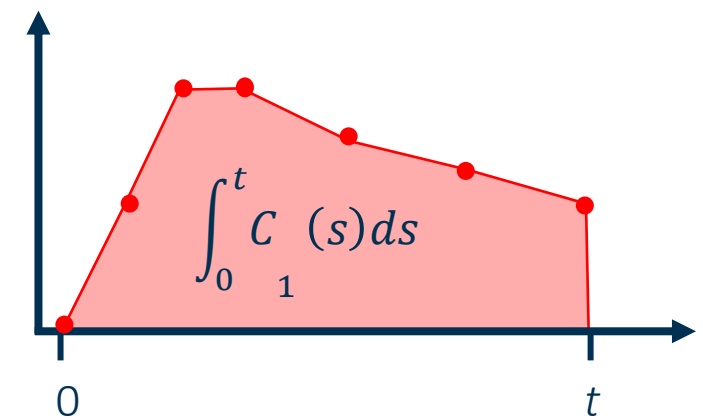
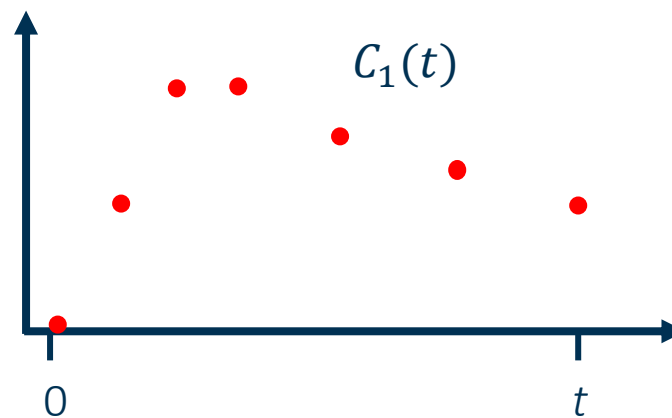
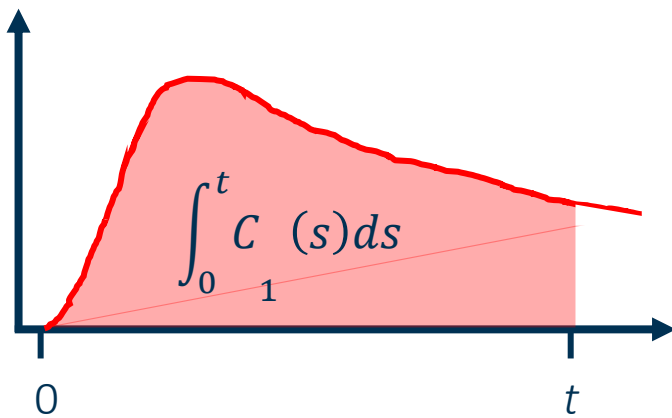
- Works also for 2 tissue compartment model



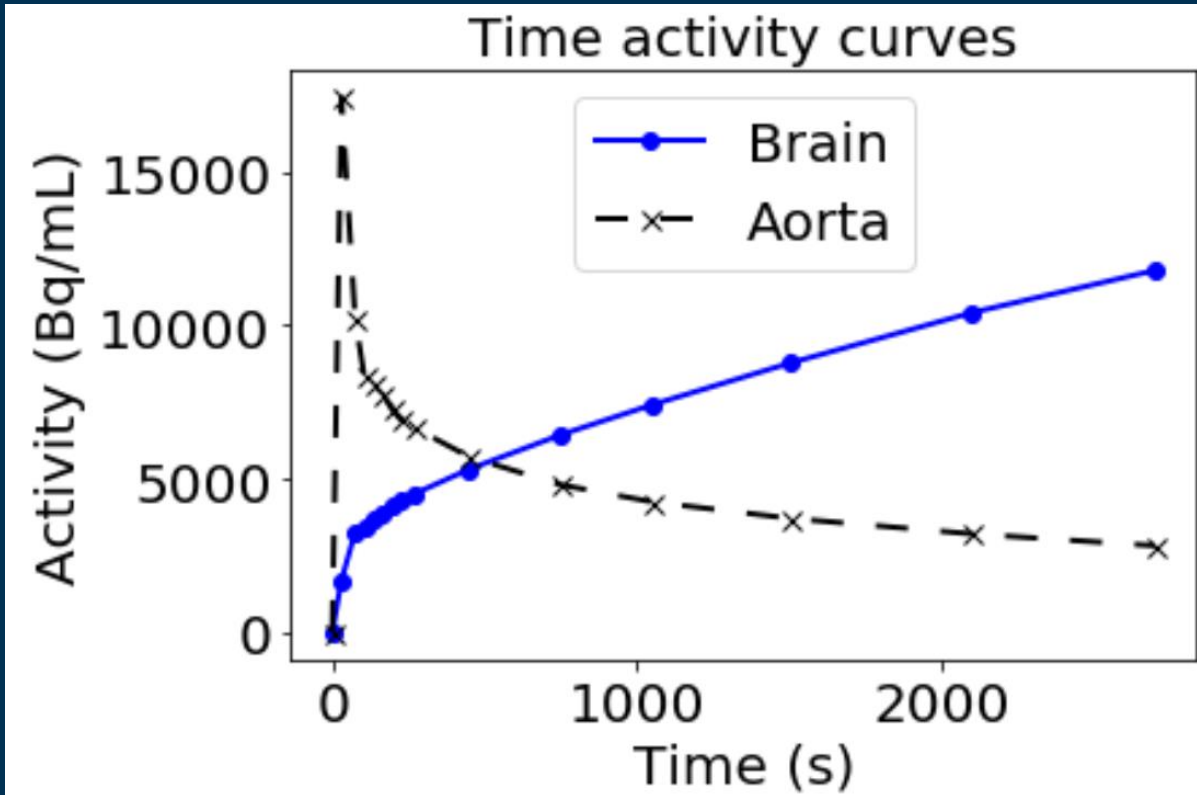
$$\frac{\partial C_2(t)}{\partial t} = C_1(t) f_1$$

What is $\int_0^t C_1(s)ds$?

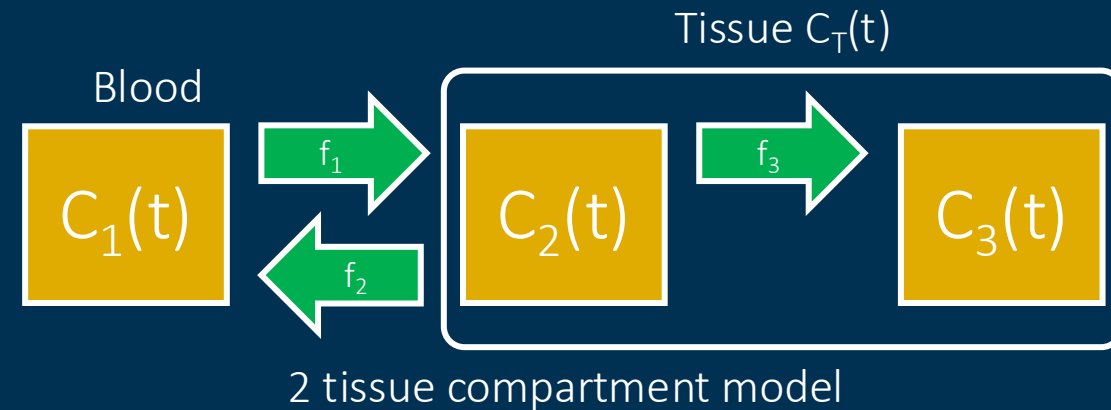
- $\int_0^t C_1$ is integral from 0 to t , which means area under the curve
- In practice simple area
 - Only rectangulars and triangles



Patlak plot for irreversible uptake, 2TCM



¹⁸F-FDG scan, brain and aorta.



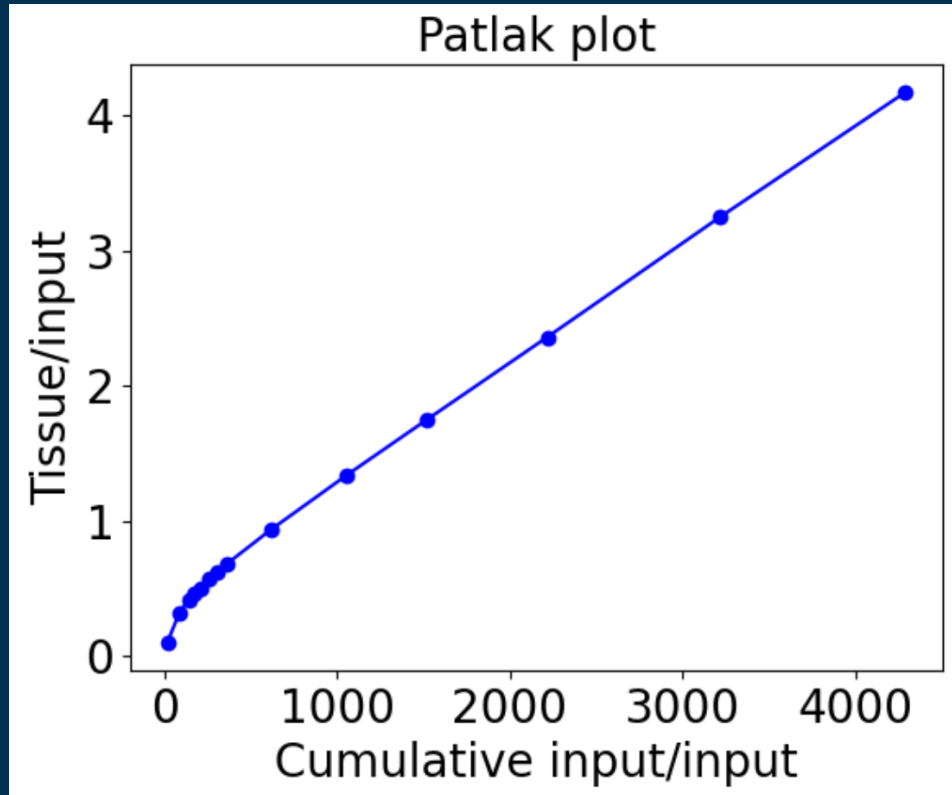
$$\frac{\partial C_1(t)}{\partial t} = C_2(t)f_2 - C_1(t)f_1$$

$$\frac{\partial C_2(t)}{\partial t} = C_1(t)f_1 - C_2(t)f_2 - C_3(t)f_3$$

$$\frac{\partial C_3(t)}{\partial t} = C_2(t)f_3$$

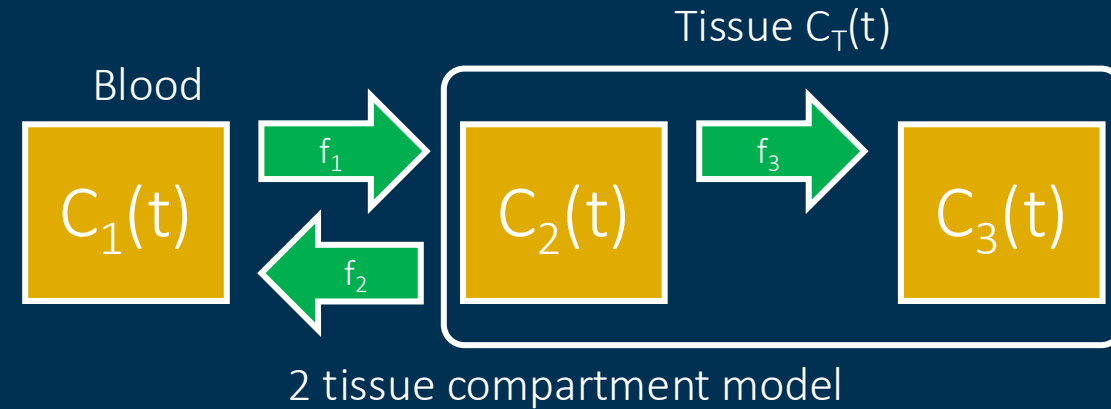
Patlak plot for irreversible uptake, 2TCM

$$\frac{C_T(t)}{C_1(t)}$$



^{18}F -FDG scan, brain and aorta.

$$\frac{\int_0^t C_1(s) ds}{C_1(s)}$$

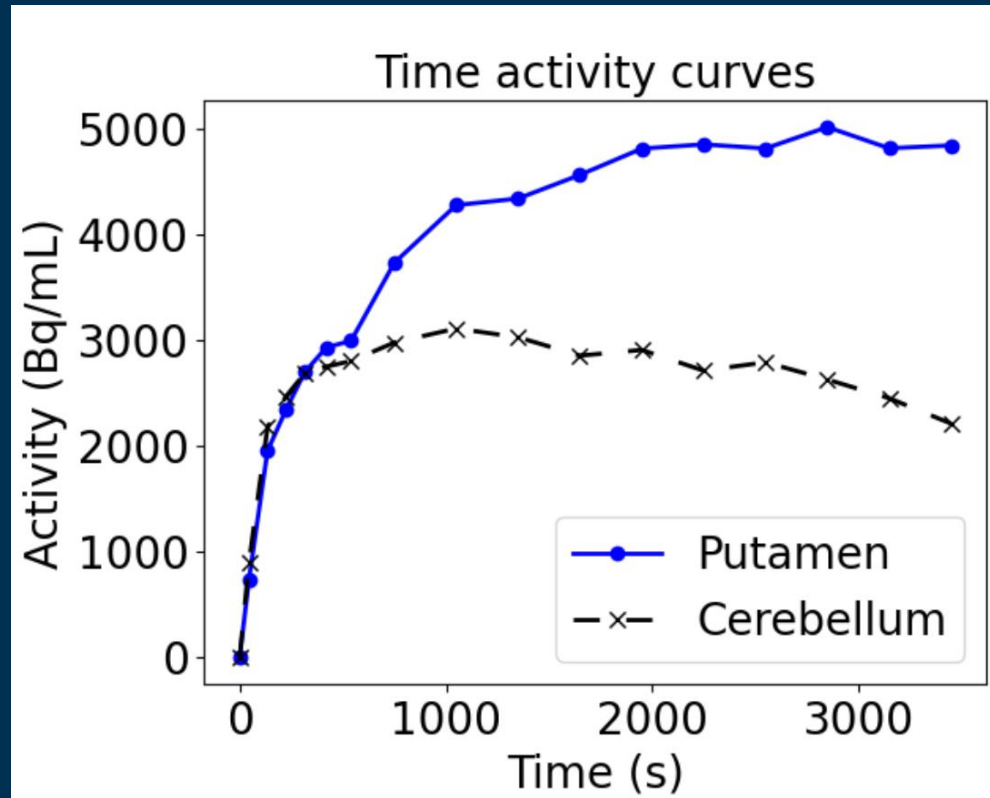


$$\frac{\partial C_1(t)}{\partial t} = C_2(t)f_2 - C_1(t)f_1$$

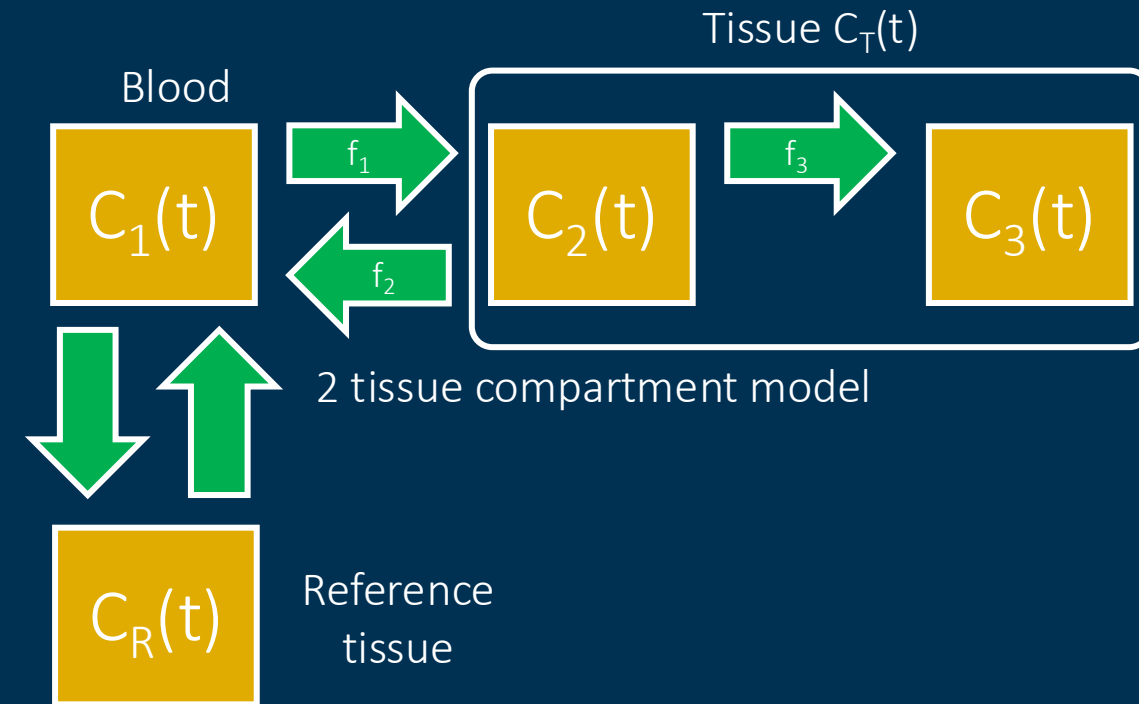
$$\frac{\partial C_2(t)}{\partial t} = C_1(t)f_1 - C_2(t)f_2 - C_3(t)f_3$$

$$\frac{\partial C_3(t)}{\partial t} = C_2(t)f_3$$

Patlak plot for irreversible uptake, 2TCM, reference tissue

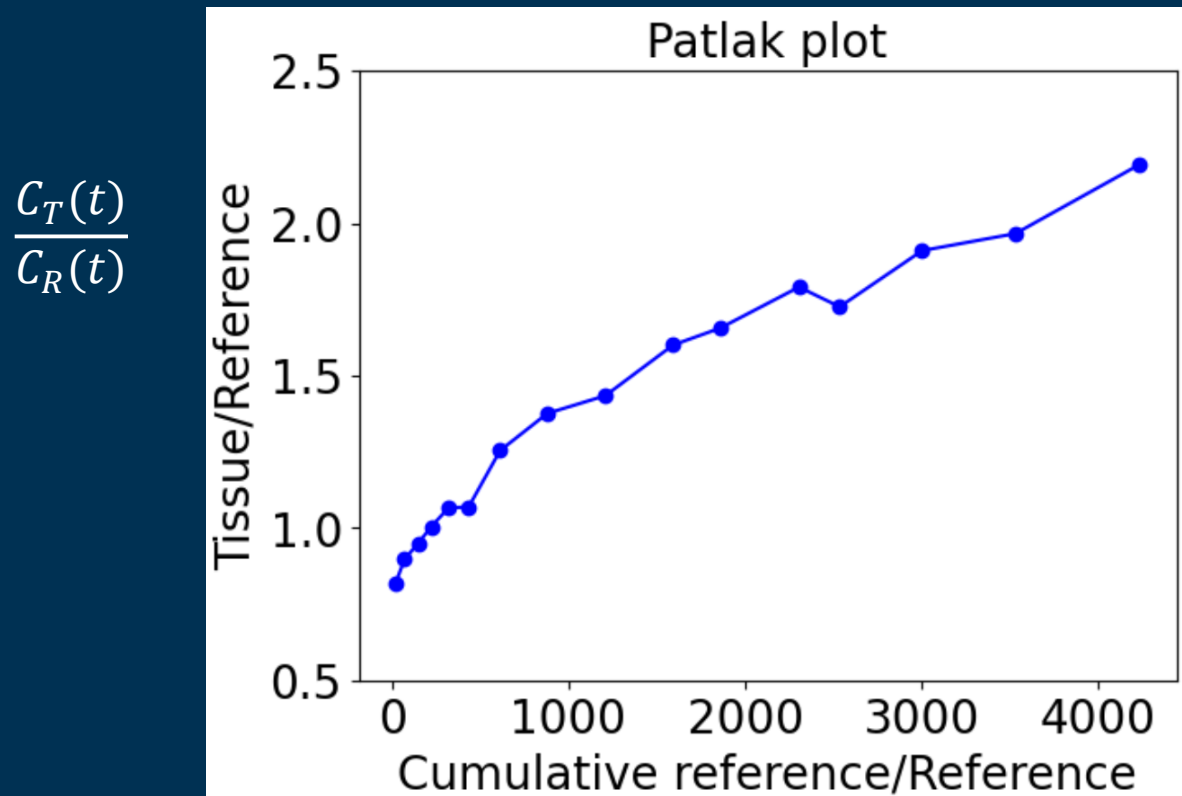


^{18}F -FDOPA scan, putamen and cerebellum.



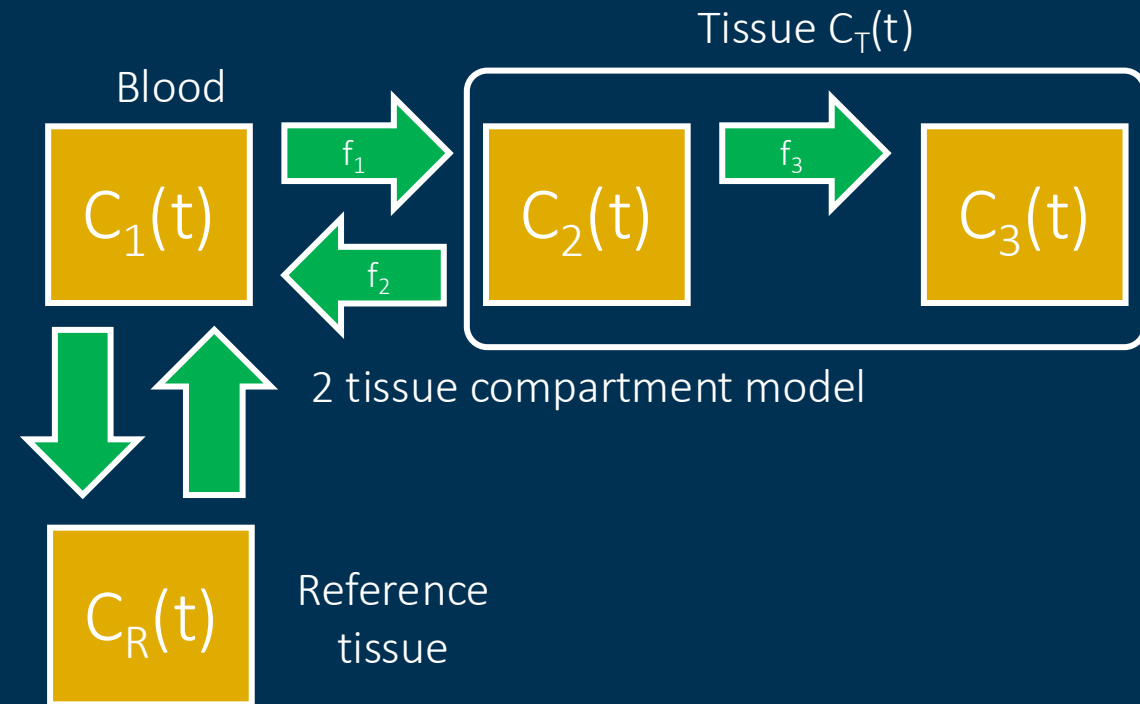
Cerebellum can be used as a reference tissue in FDG or FDOPA studies.

Patlak plot for irreversible uptake, 2TCM, reference tissue



^{18}F -FDOPA scan, putamen and cerebellum.

$$\frac{\int_0^t C_R(s) ds}{C_R(s)}$$



Cerebellum can be used as a reference tissue in FDG or FDOPA studies.

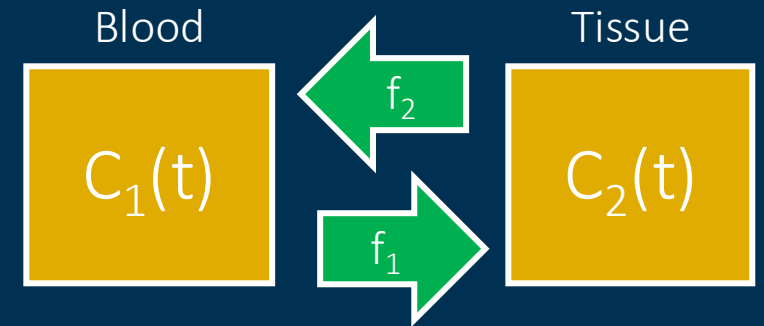
Logan plot for reversible uptake, 1TCM

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In other words, plotting $\frac{\int_0^t C_2(s) ds}{C_2(t)}$ against $\frac{\int_0^t C_1(s) ds}{C_2(t)}$ gives a line

- Works also for 2 tissue compartment model

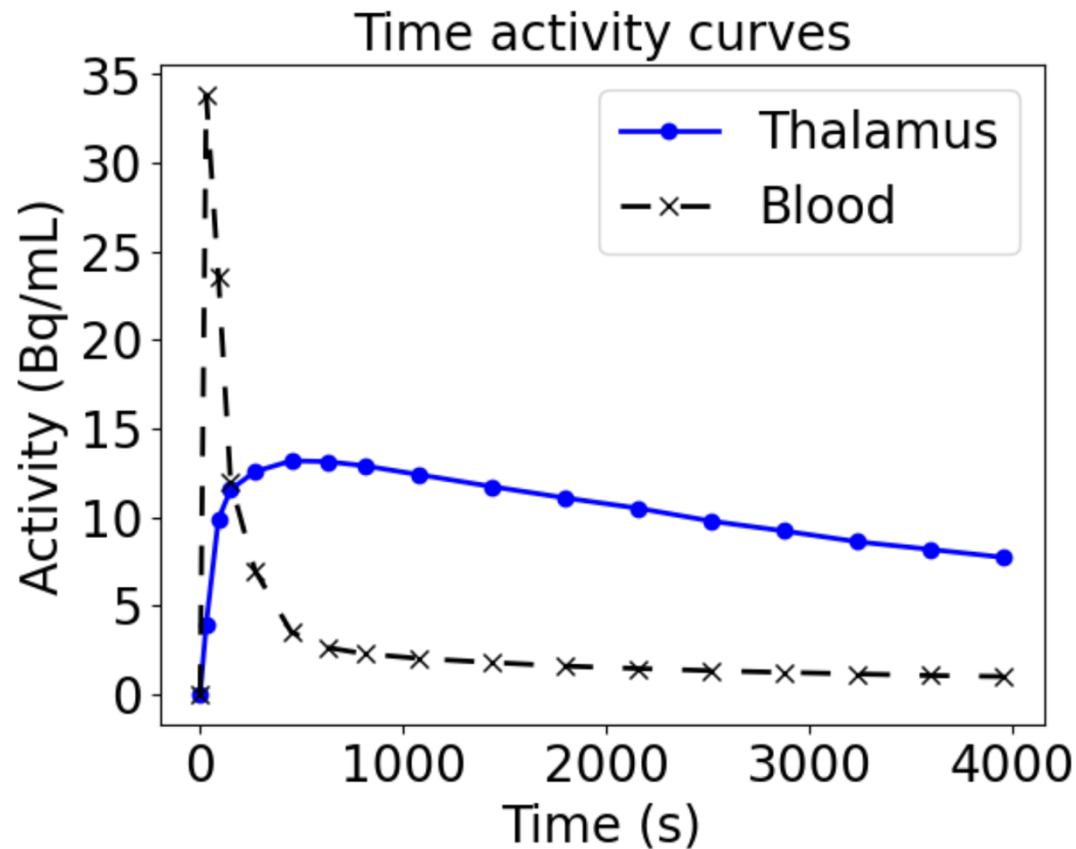


1 tissue compartment model

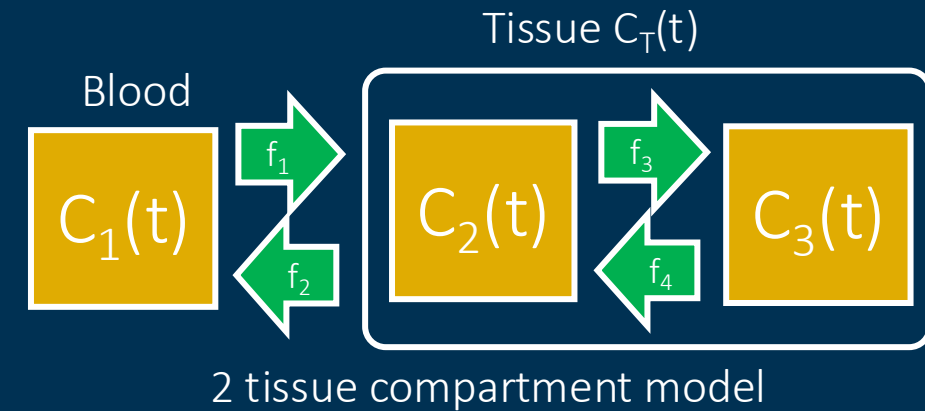
$$\frac{\partial C_1(t)}{\partial t} = C_2(t)f_2 - C_1(t)f_1$$

$$\frac{\partial C_2(t)}{\partial t} = C_1(t)f_1 - C_2(t)f_2$$

Logan plot for reversible uptake, 2TCM



¹¹C-carfentanil scan, thalamus and blood.

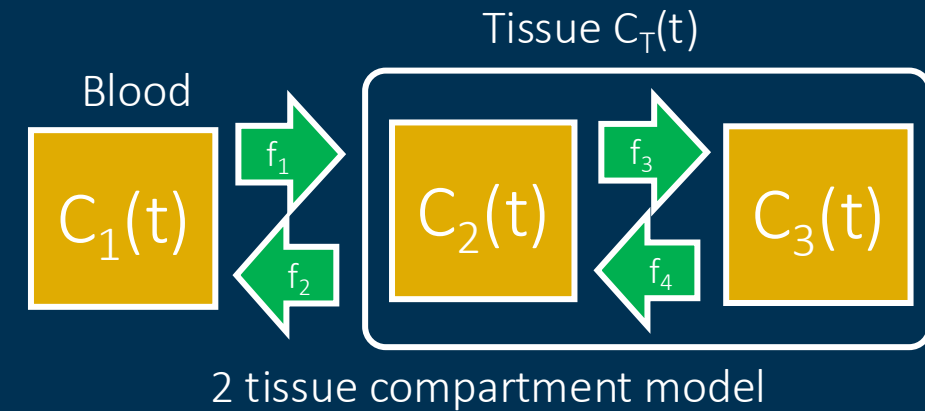
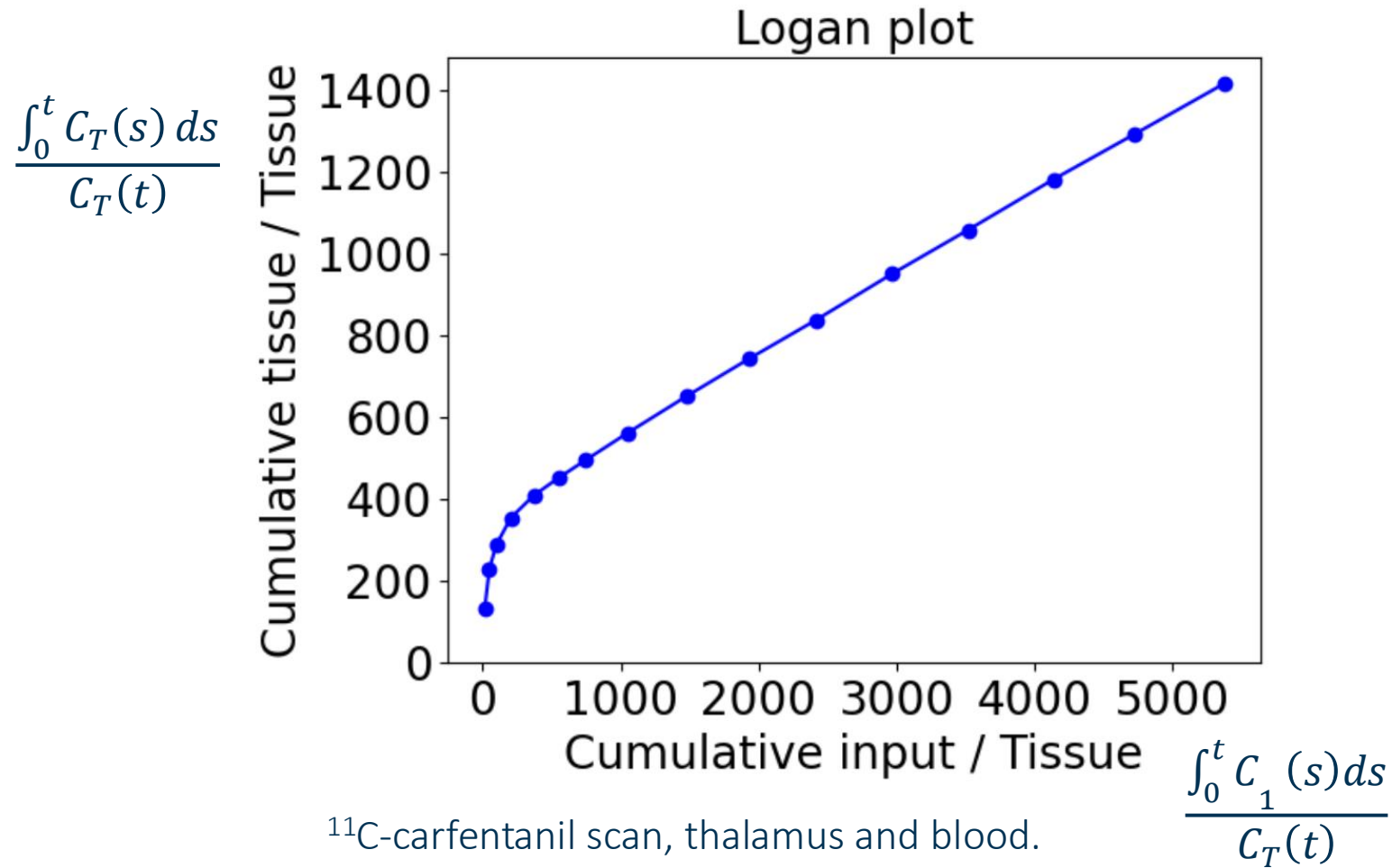


$$\frac{\partial C_1(t)}{\partial t} = C_2(t)f_2 - C_1(t)f_1$$

$$\frac{\partial C_2(t)}{\partial t} = C_1(t)f_1 - C_2(t)(f_2 + f_3) + C_3(t)f_4$$

$$\frac{\partial C_3(t)}{\partial t} = C_2(t)f_3 - C_3(t)f_4$$

Logan plot for reversible uptake, 2TCM

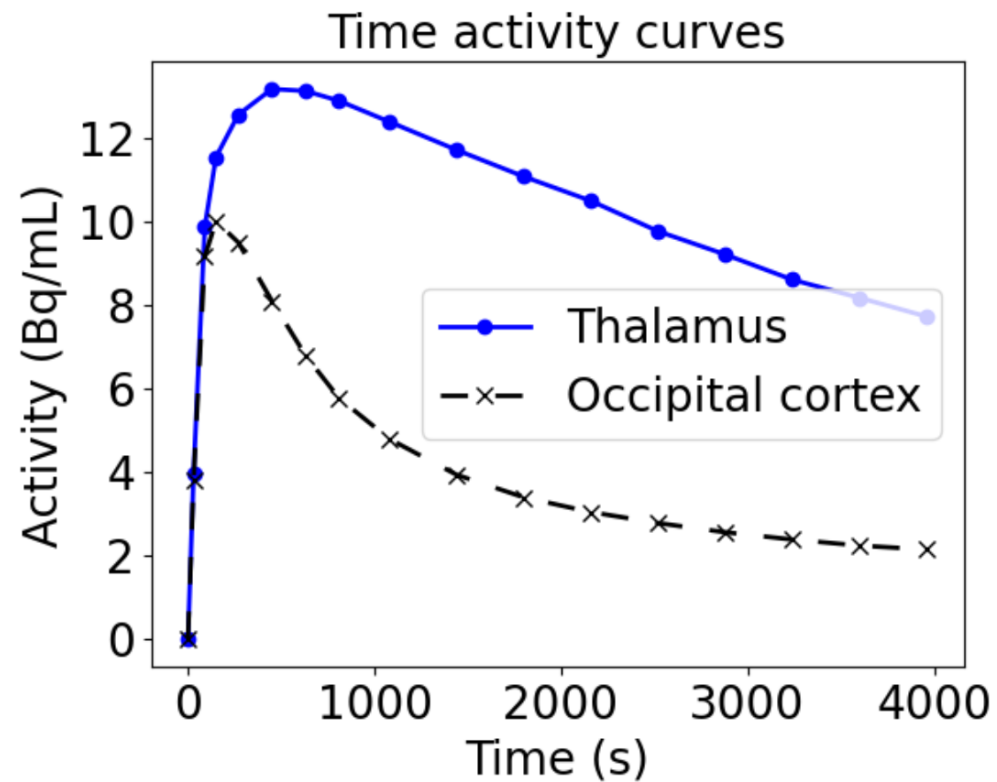


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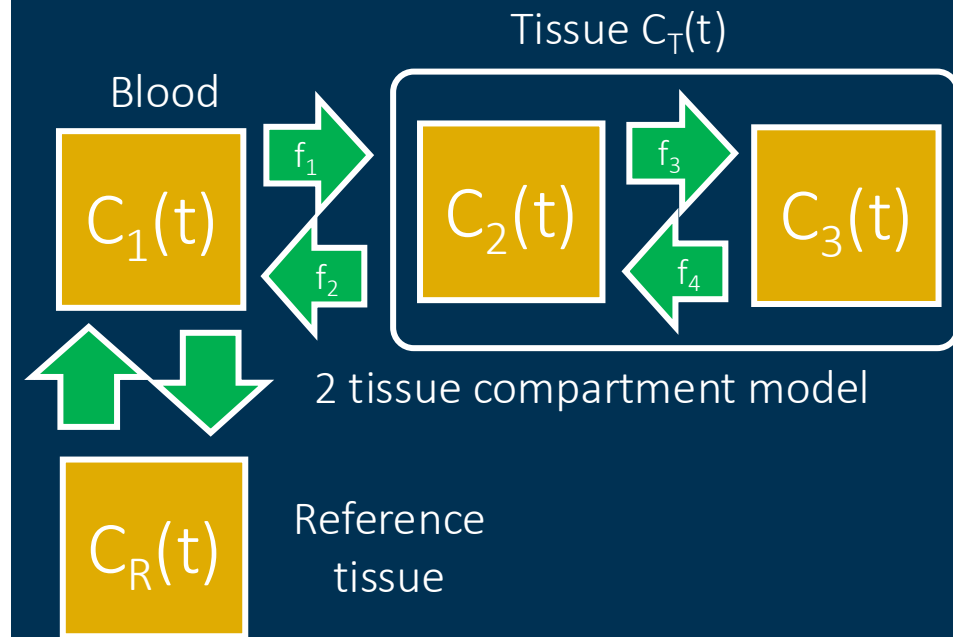
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$$\frac{\partial C_3(t)}{\partial t} = C_2(t)f_3 - C_3(t)f_4$$

Logan plot for reversible uptake, 2TCM, reference tissue

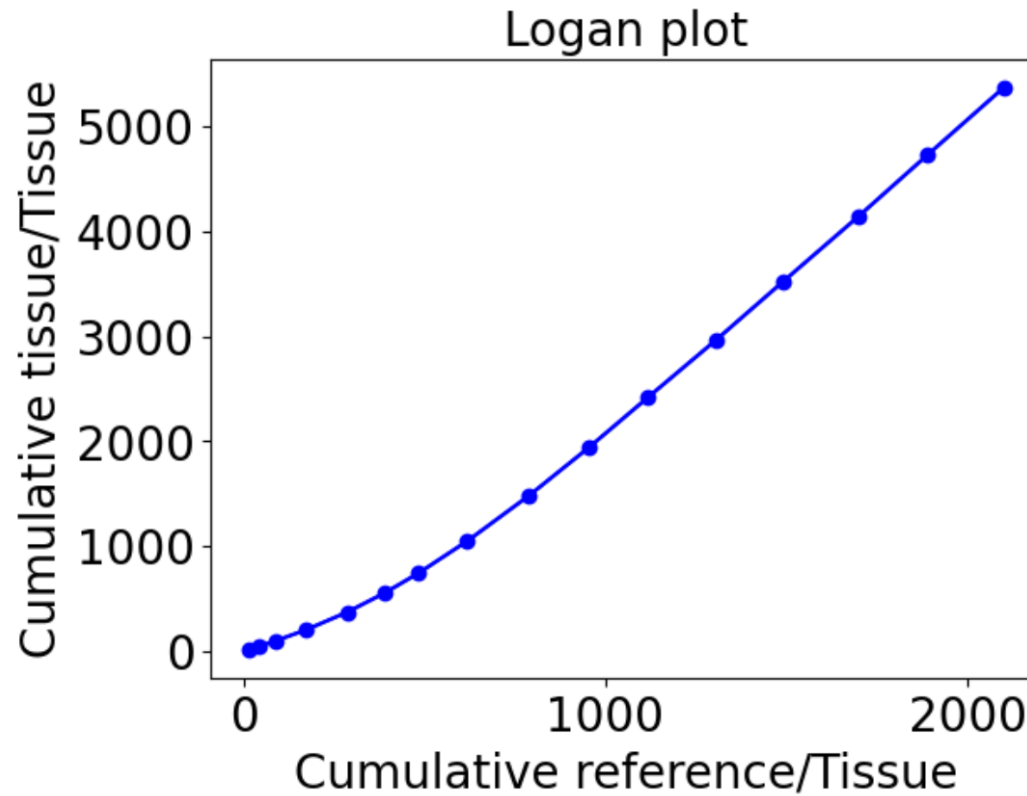


^{11}C -carfentanil scan, thalamus and occipital cortex.

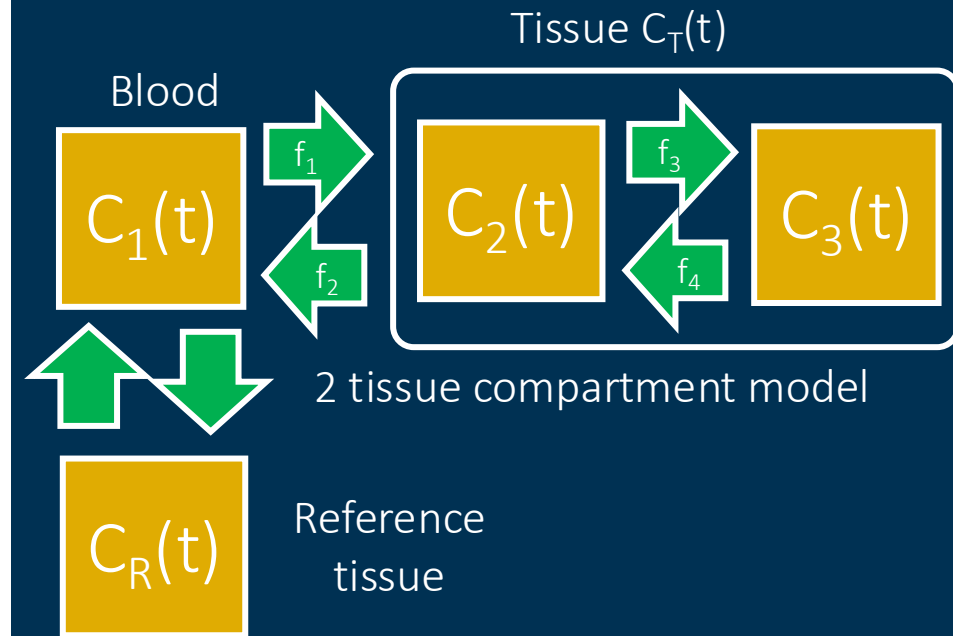


Logan plot for reversible uptake, 2TCM, reference tissue

$$\frac{\int_0^t C_T(s) ds}{T_T(t)}$$

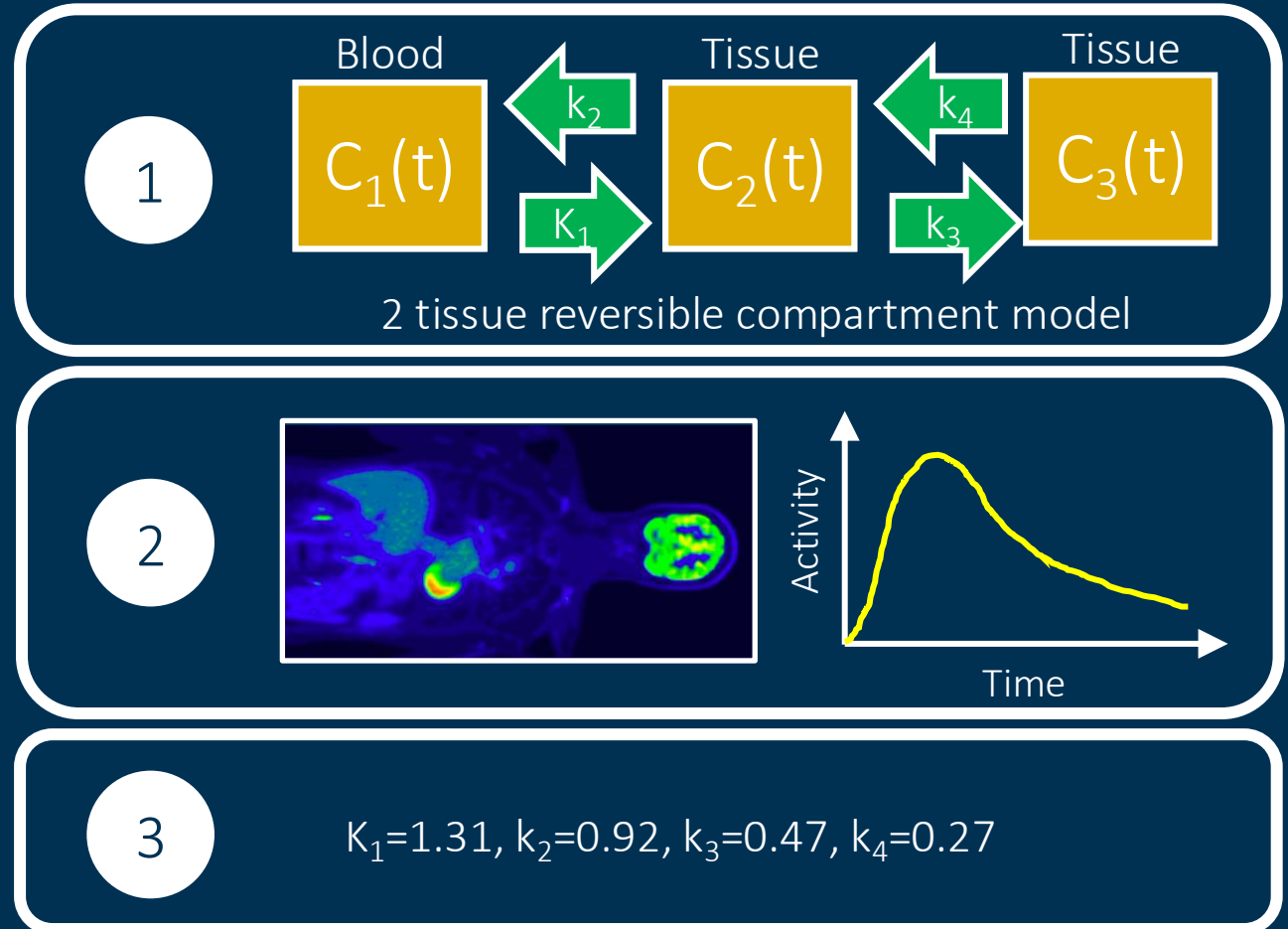


$$\frac{\int_0^t C_1(s) ds}{C_T(t)}$$



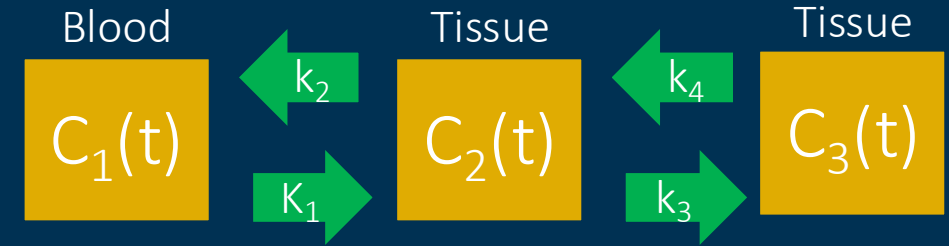
Compartment models in PET

- Step one: define your model
 - Based on literature or experiments
- Step two: obtain data
 - PET scan and image analysis
- Step three: modelling
 - Use software to find coefficients



How to interpret modelling results?

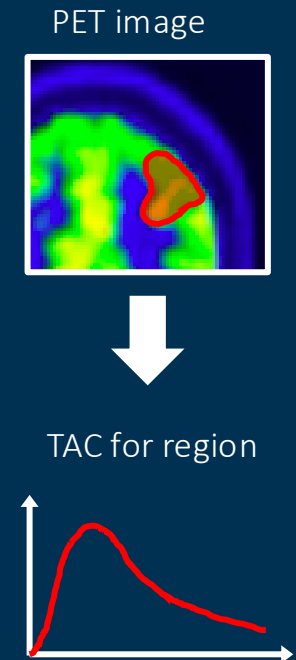
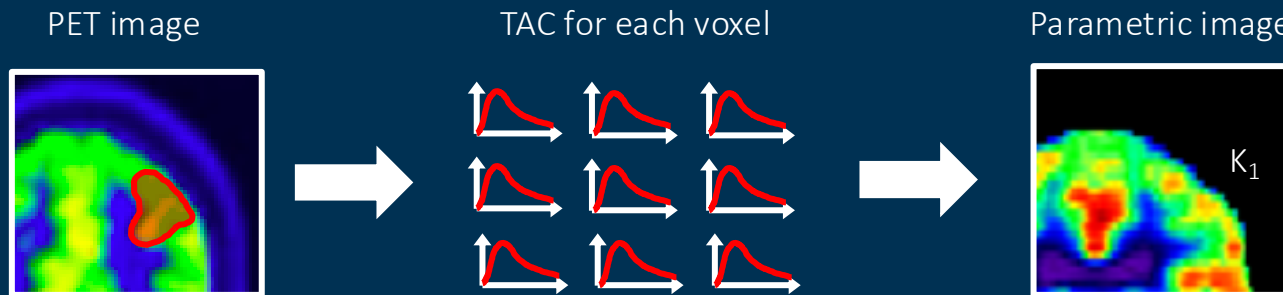
- Interpretation depends on the tracer
 - Different tracers are used to image different functions
- For example, FDG accumulates in myocardium and is used to image glucose uptake
 - FDG can be used to measure metabolism
 - Cancer tissue can be located with FDG
- In many applications the most interesting parameter is K_1
 - It represents the transport of the tracer blood compartment to the first tissue compartment
 - Larger value means faster transport



2 tissue reversible compartment model

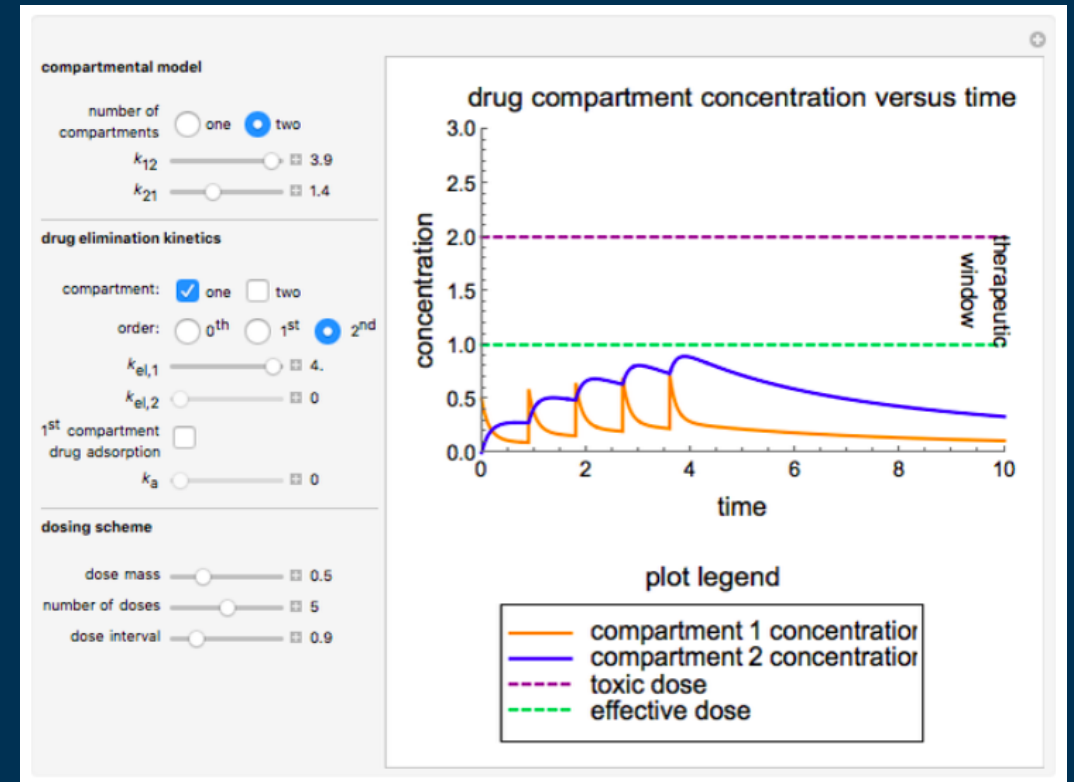
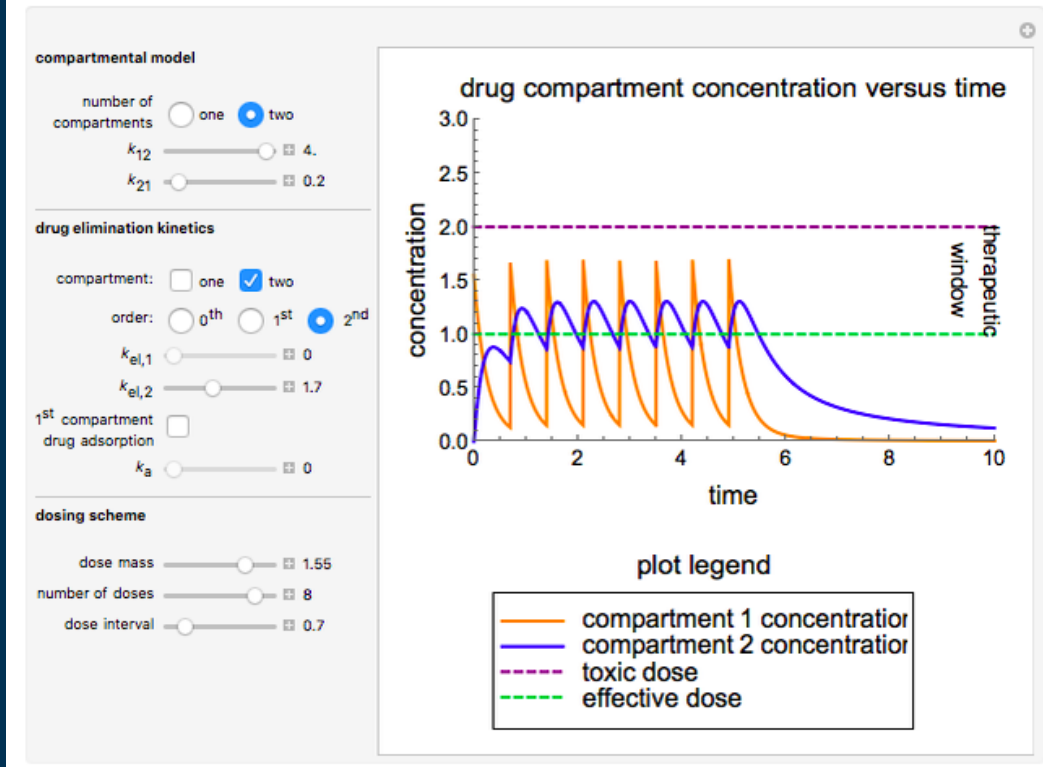
Parametric imaging

- For modelling regions, only one activity value per time point is considered
- Parametric imaging refers to modelling of individual voxels
 - Compartment modelling computationally demanding
 - Typically Logan or Patlak methods are used



Example MATLAB project compartment models

Pharmacokinetic Modeling



<https://demonstrations.wolfram.com/PharmacokineticModeling/>



What you should have learn

Contents of the talk

- PET imaging
- Image segmentation
- PET modelling
- Interpretation

Key terminology

- Dynamic PET
- TAC
- Compartment model
- Patlak plot
- Logan plot
- Parametric imaging



Forum for Interdisciplinary Mathematics

Riku Klén · Oona Rainio

Modeller's Guide to PET Imaging

Kinetic modelling in positron emission tomography (PET) has evolved alongside scanner development over the past 50 years. Yet, despite its central role in quantitative analysis, a single, comprehensive resource covering both theory and applications has been missing. Building on decades of expertise at Turku PET Centre, Modeller's Guide to PET Imaging brings this knowledge together in a clear, systematic textbook.

Designed for clinicians, physicists, and researchers alike, this book offers a structured introduction to the analysis of dynamic PET data. It combines theoretical foundations with practical implementation, explaining mathematical concepts, statistical methods, and modelling techniques through accessible text, illustrations, and fully worked Python examples.

The book spans the full workflow of PET analysis—from imaging physics and data processing to advanced kinetic modelling. Core topics include compartment models, graphical approaches such as Patlak and Logan plots, and spectral analysis, alongside applications across major organs.

With more than 120 exercises and extensive code examples, readers are encouraged to apply concepts directly to real PET data, making this both a practical learning resource and a long-term reference.



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Modeller's Guide to PET Imaging

Forum for Interdisciplinary Mathematics

Riku Klén
Oona Rainio

Modeller's Guide to PET Imaging



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