# GENERAL LINEAR MODEL FOR FMRI ANALYSIS

Turku PET Centre Brain Imaging Course 2025

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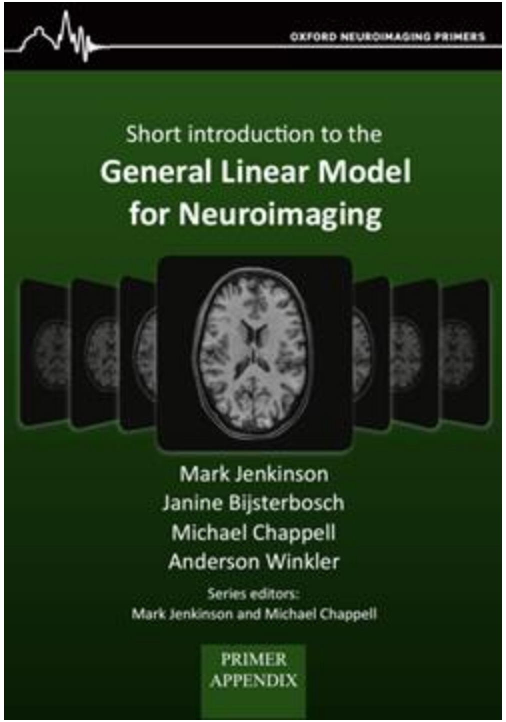
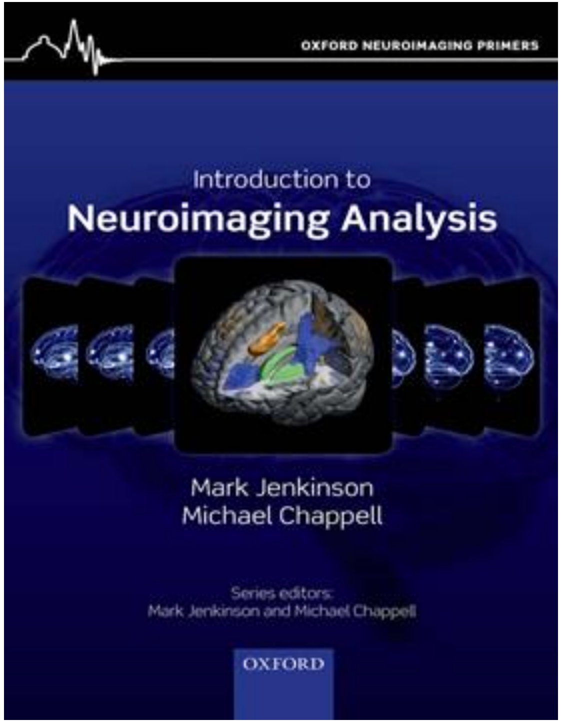
# Resources

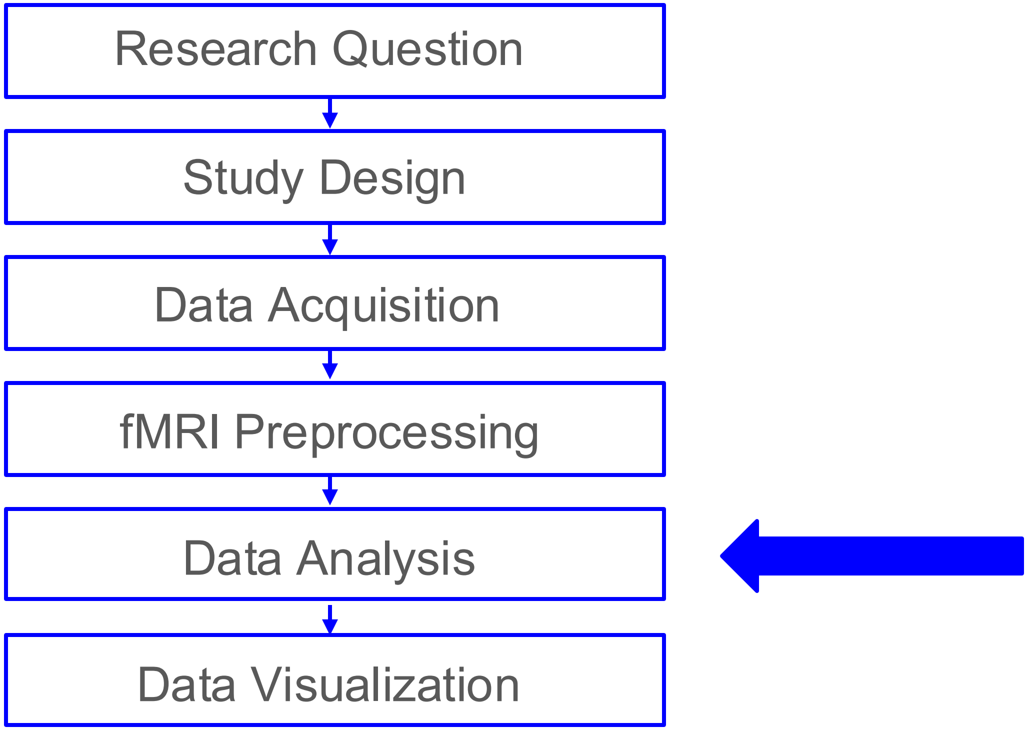



YouTube FI

[Principles of fMRI Part 1 Module 15: The General Linear Model - Intro]

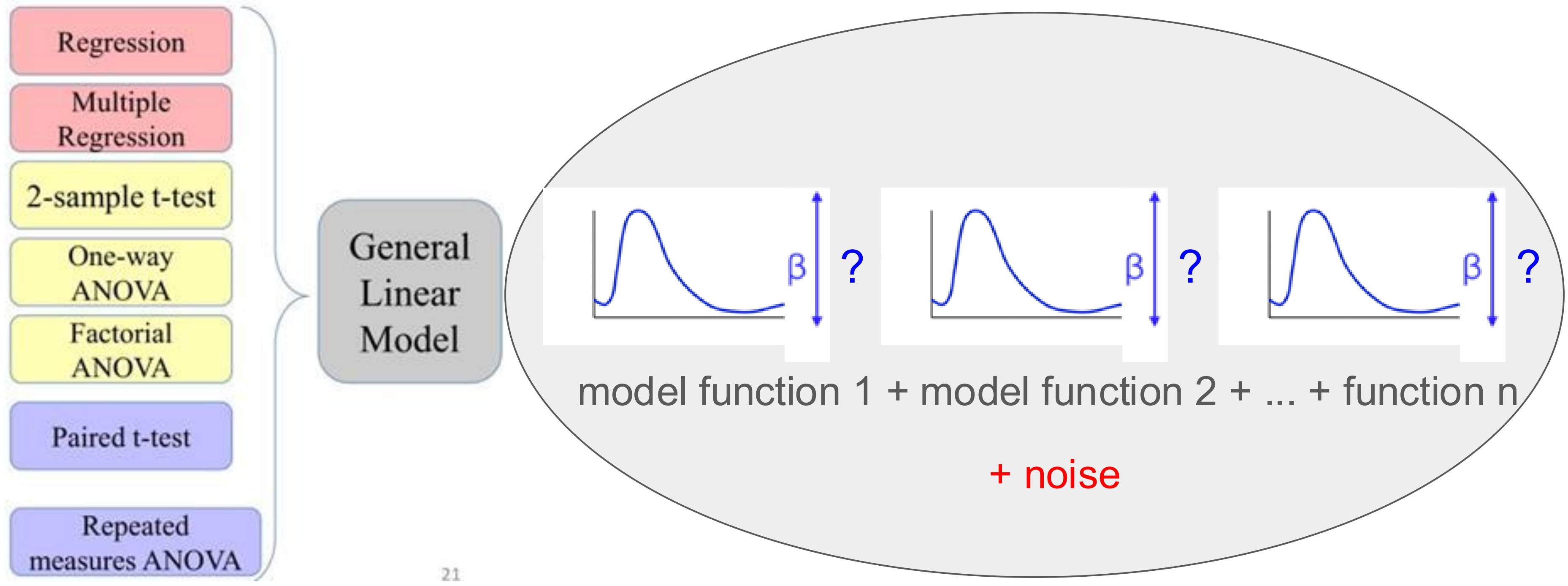
[Principles of fMRI Part 1, Module 16: GLM applied to fMRI]


Research Question
Study Design
Data Acquisition
fMRI Preprocessing
Data Analysis
Data Visualization

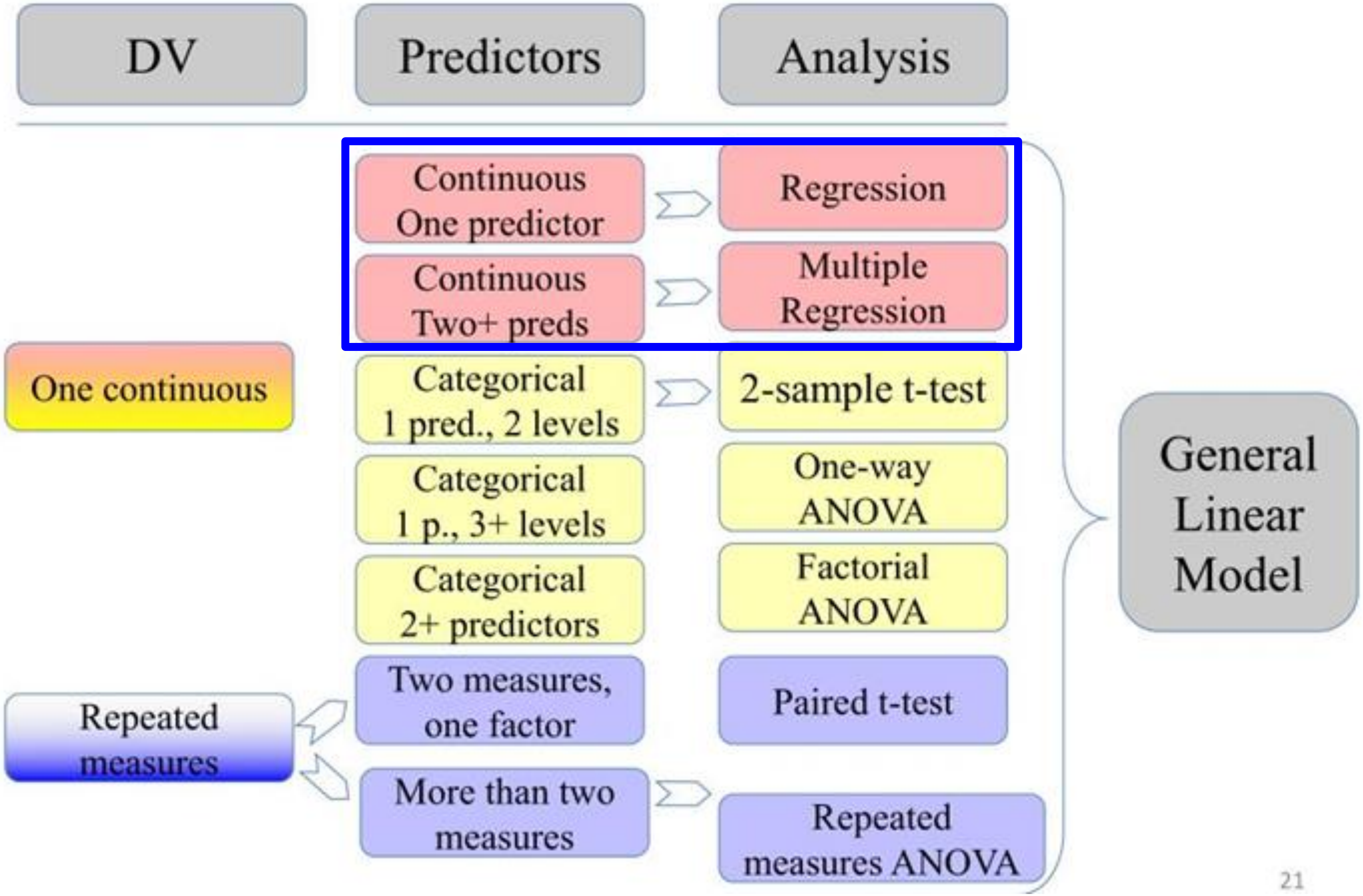
# Outline

1. General Linear Model (GLM) family
2. GLM for fMRI Analysis
    a. Simple Regression
    b. Multiple Regression
    c. Covariates and Confounds
    d. Limitations
3. Case Study & Quiz

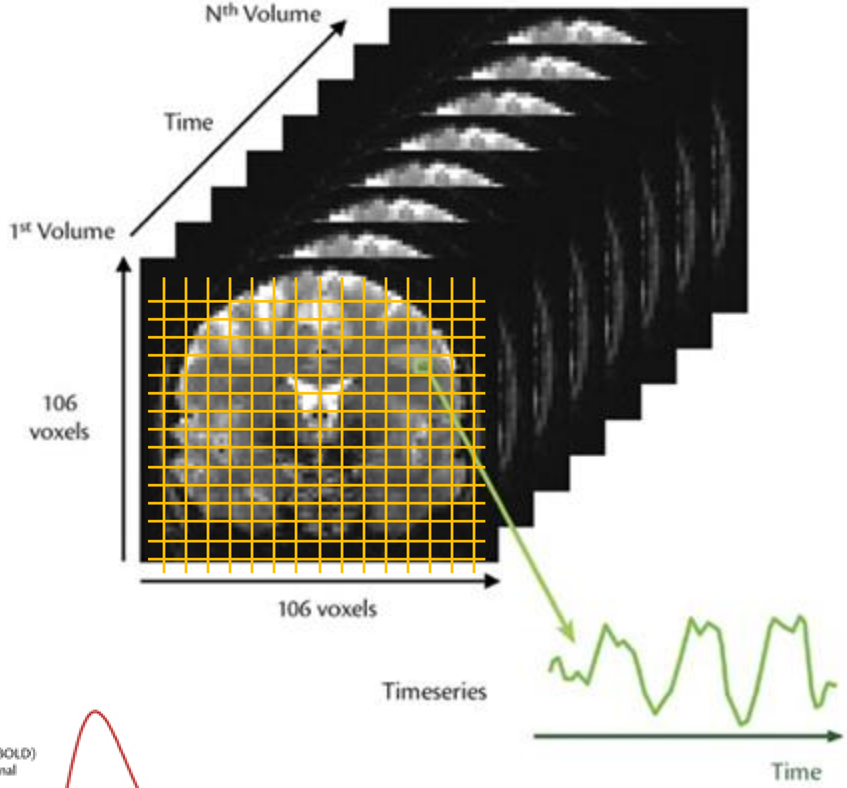
# GLM family


Regression
Multiple Regression
2-sample t-test
One-way ANOVA
Factorial ANOVA
Paired t-test
Repeated measures ANOVA
General Linear Model
21
β
?
β
?
β
?
model function 1 + model function 2 + ... + function n
+ noise

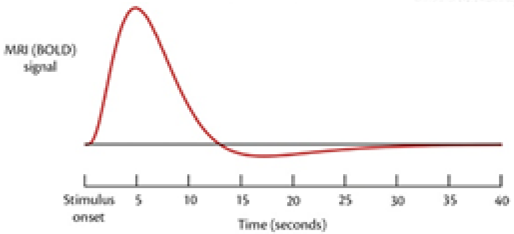
# GLM family


DV
Predictors
Analysis
Continuous
One predictor
Regression
Continuous
Two+ preds
Multiple
Regression
One continuous
Categorical
1 pred., 2 levels
2-sample t-test
Categorical
1 p., 3+ levels
One-way
ANOVA
Categorical
2+ predictors
Factorial
ANOVA
Repeated
measures
Two measures,
one factor
Paired t-test
More than two
measures
Repeated
measures ANOVA
General
Linear
Model

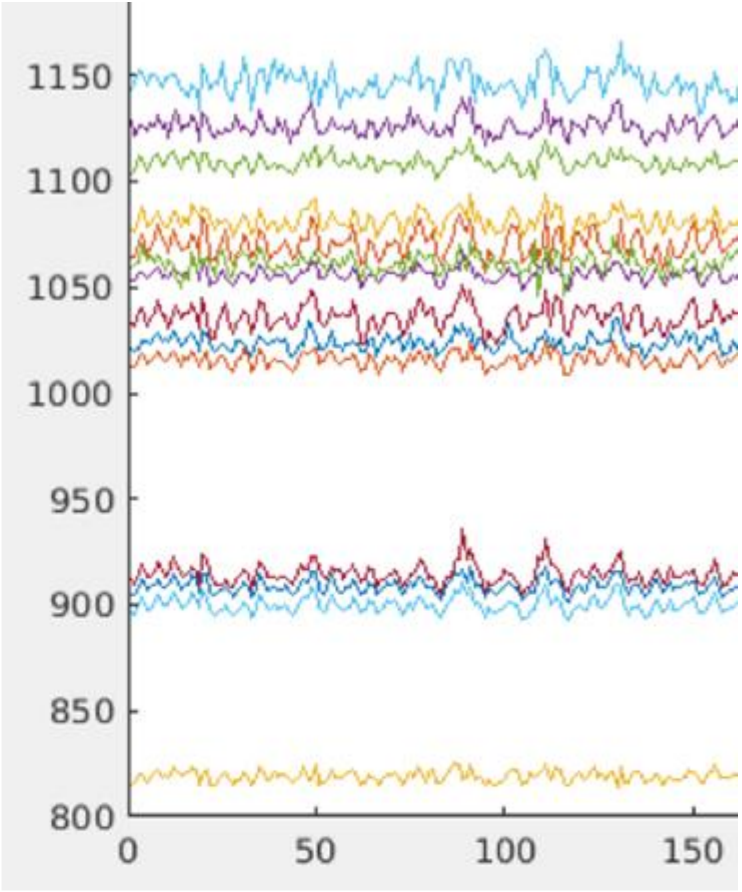
# Reminder



Nth Volume
Time
1st Volume
106 voxels
106 voxels
Timeseries
Time

MRI (BOLD) signal
Stimulus onset
5
10
15
20
25
30
35
40
Time (seconds)

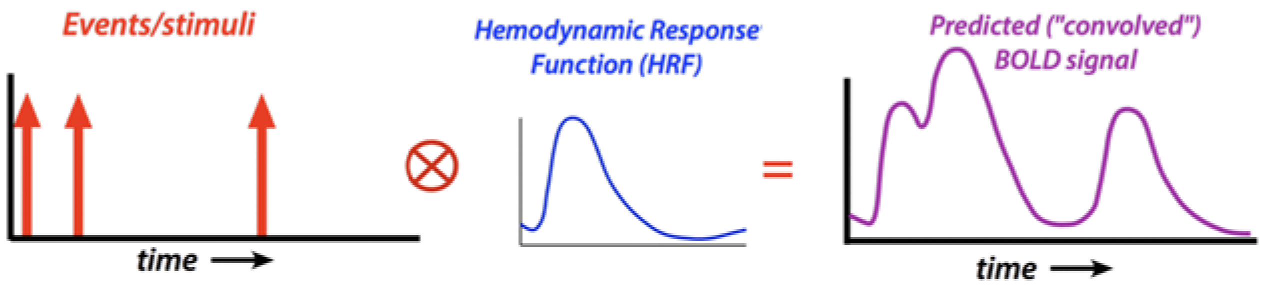
1150
1100
1050
1000
950
900
850
800
0
50
100
150

# GLM for fMRI Analysis

GLM

- looks for the predicted responses in the measured fMRI signal.

# Predicted response model




Boxcar stimulus timing

Predicted response

Event-related stimulus timing

Predicted response

# GLM for fMRI Analysis

GLM

- looks for the predicted responses in the measured fMRI signal.

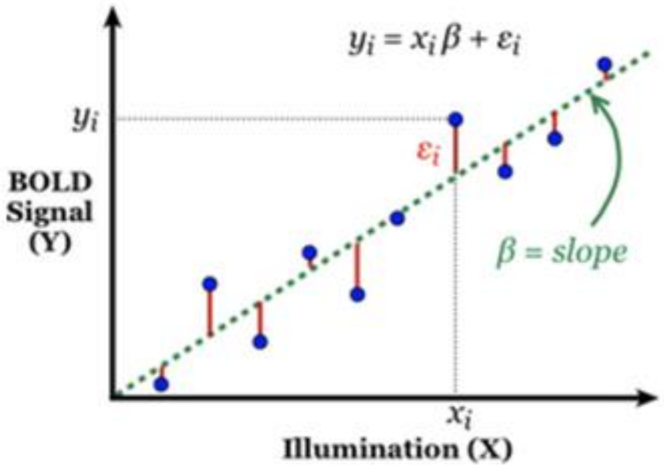
- helps determine where there is significant relationship between the measured signal and the predicted response

# Simple Regression

$$Y = X\beta + \varepsilon$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} [\beta] + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

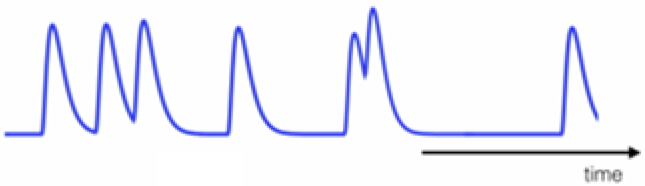
- Y = BOLD timeseries
- X = regressor
- β = scaling parameter
- ε = residual error / residuals

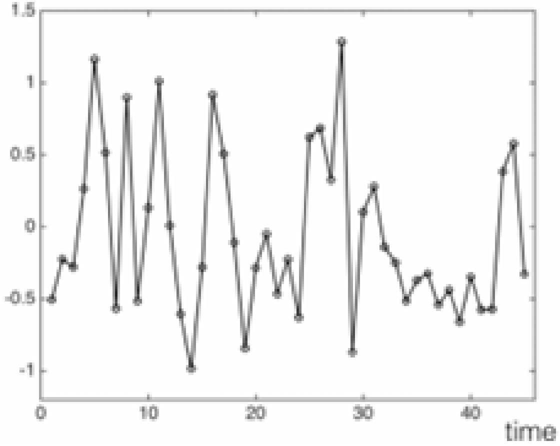



Simplified linear regression example, adapted from data of Hansen et al (2004)

# Simple Regression

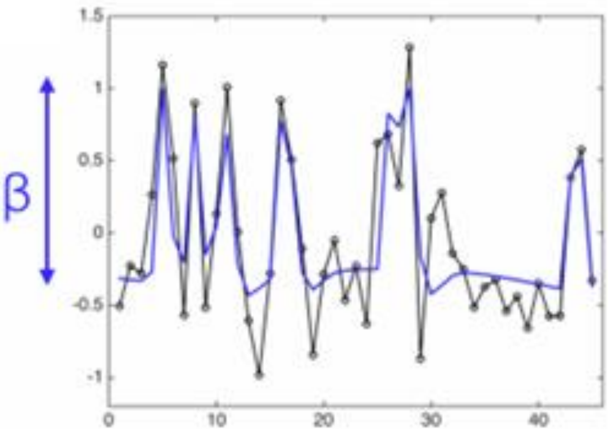
predicted response

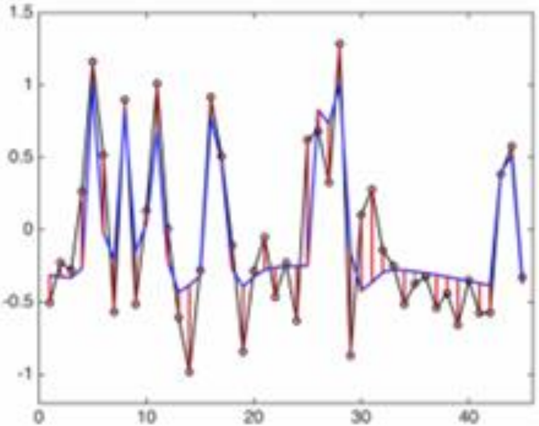



BOLD timeseries

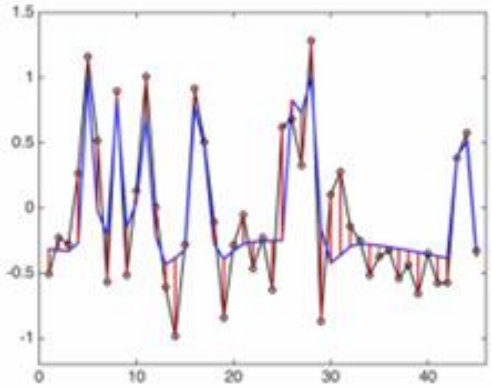
GLM fitting




$$Y = X\beta + \varepsilon$$

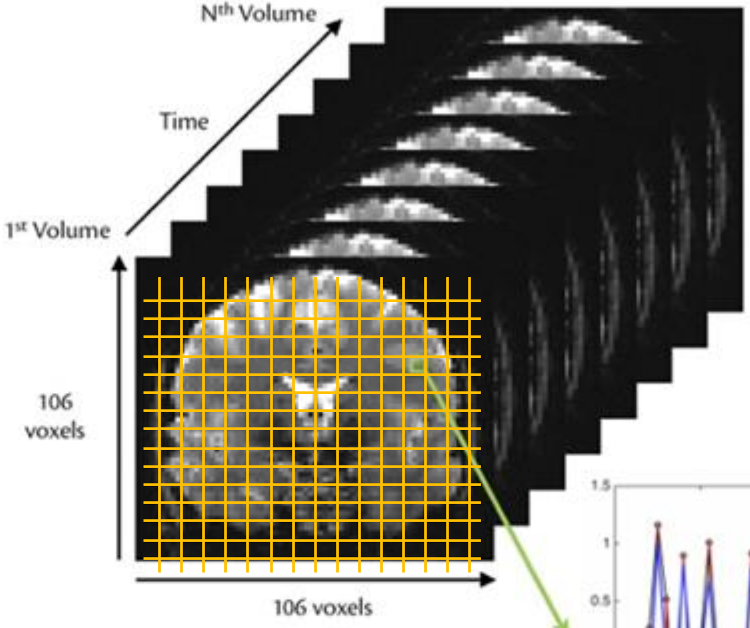
# “Best fit”

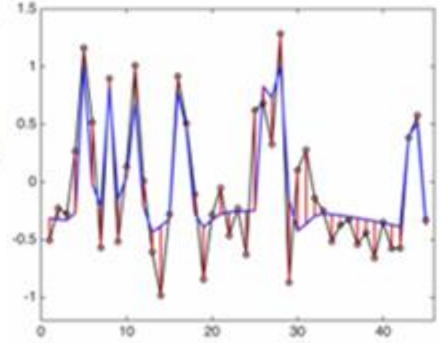



Difference between the residuals are minimised

→ Minimising the residuals / Finding the least squared error

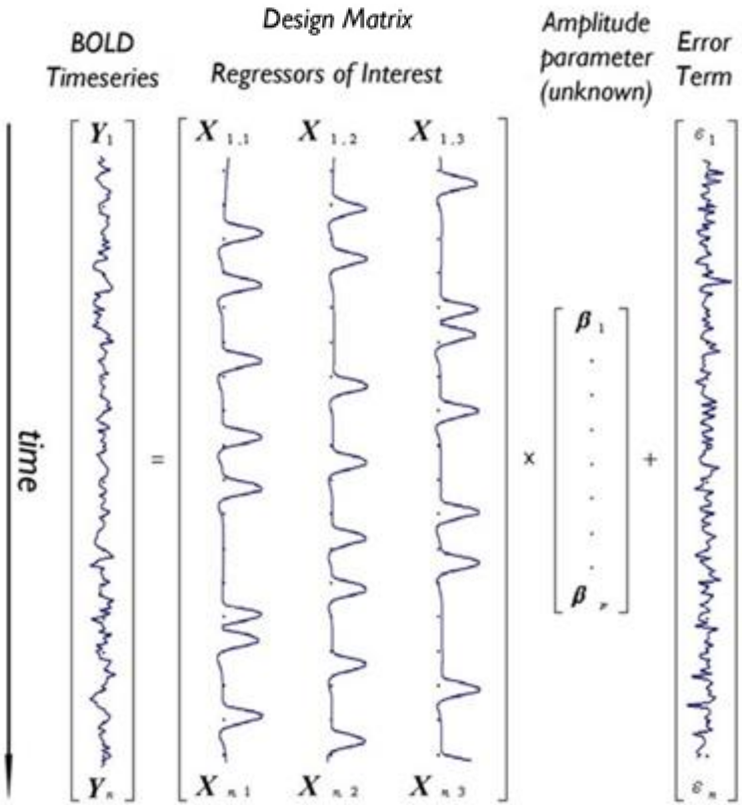
# Voxelwise + Mass Univariate Analysis




$$\begin{aligned} y_1 &= x_1\beta + \varepsilon_1 \\ y_2 &= x_2\beta + \varepsilon_2 \\ y_3 &= x_3\beta + \varepsilon_3 \\ &\vdots \\ y_n &= x_n\beta + \varepsilon_n \end{aligned} \quad \longrightarrow \quad \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} [\beta] + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

# Multiple Regression



# Multiple Regression

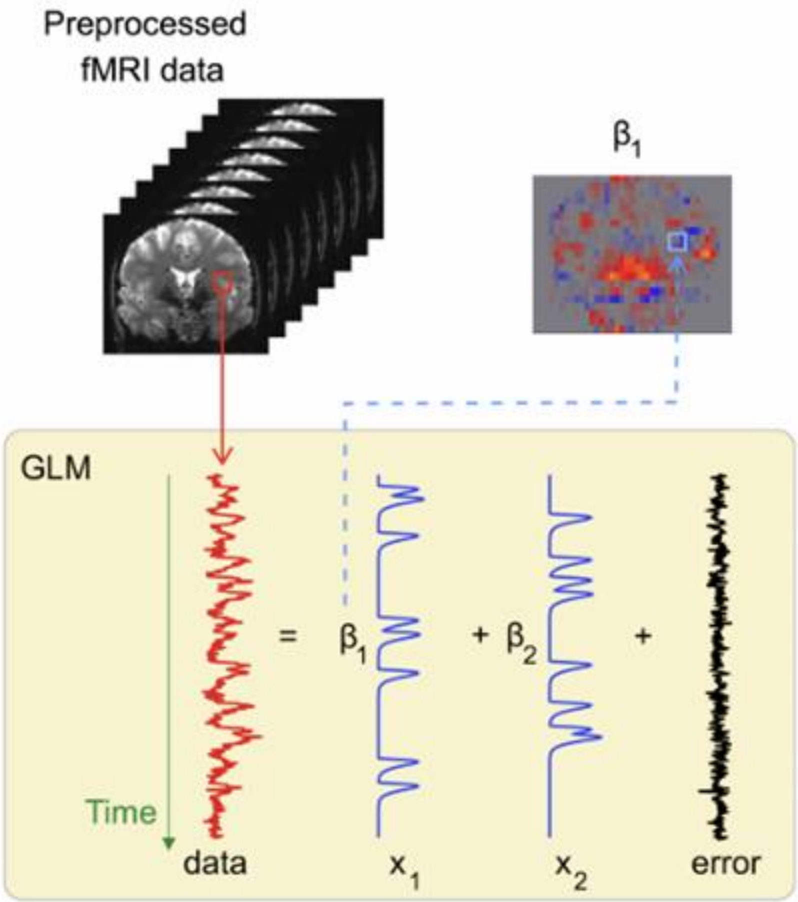
$$Y = X\beta + \varepsilon$$

time

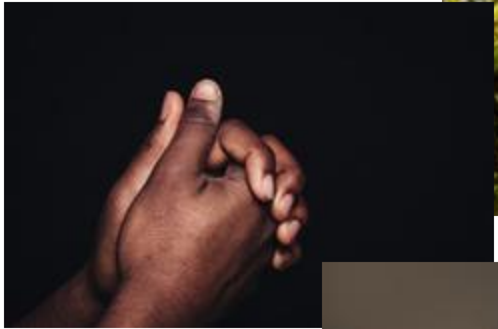
$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1p} \\ 1 & X_{21} & \cdots & X_{2p} \\ \vdots & \vdots & & \vdots \\ 1 & X_{np} & \cdots & X_{np} \end{bmatrix} \times \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

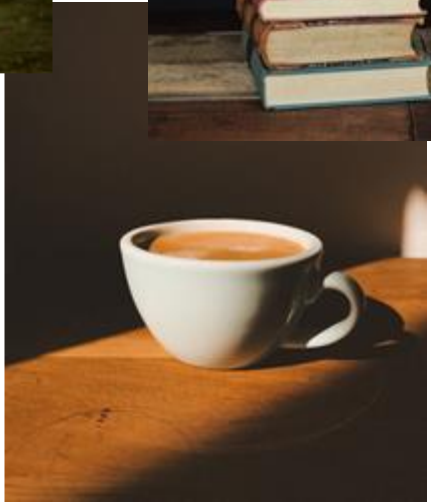
- Y = BOLD timeseries
- X = design matrix of all the regressors
  each regressor = one column
- β = vector of all the individual scaling parameters
- ε = residual error / residuals

# Results

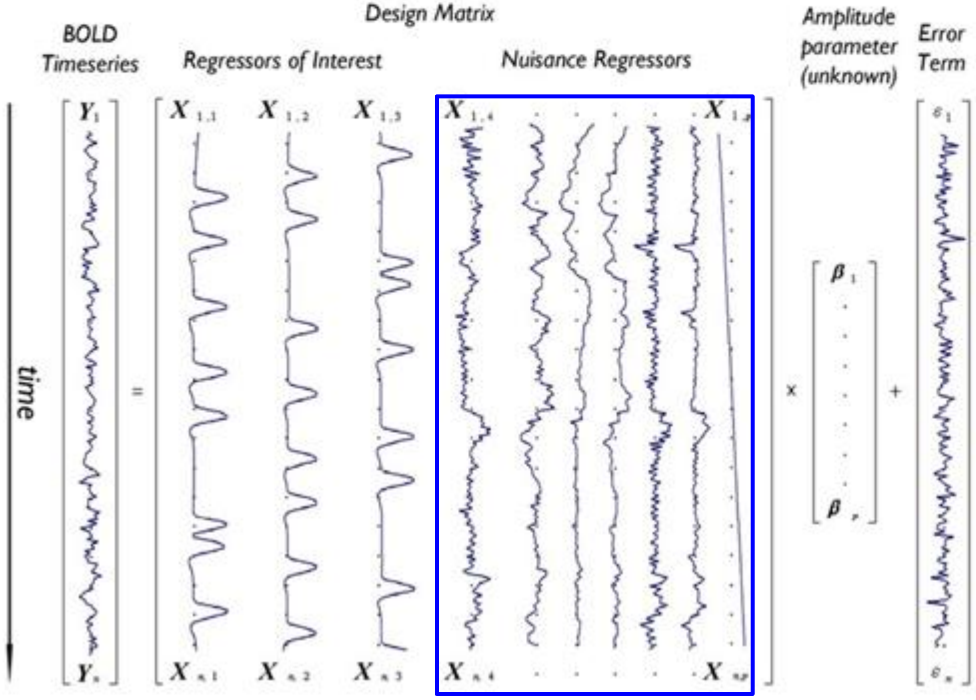


# Covariates of (no) Interests





# Confounds / Nuisance Regressors




Depiction of the General Linear Model (GLM) for a voxel with time-series **Y** predicted by a design matrix **X** including 10 effects (three regressors of interest – e.g., tasks A,B,C – and seven nuisance regressors – e.g., six motion parameters and one linear drift). Calculated weighting factors ($\beta_1$ – $\beta_{10}$) corresponding to each regressor are placed in amplitude vector **β** while column vector **ε** contains calculated error terms ($\varepsilon_i$) for the model corresponding to each time point *i*. (From Monti, 2011, under CC BY license)

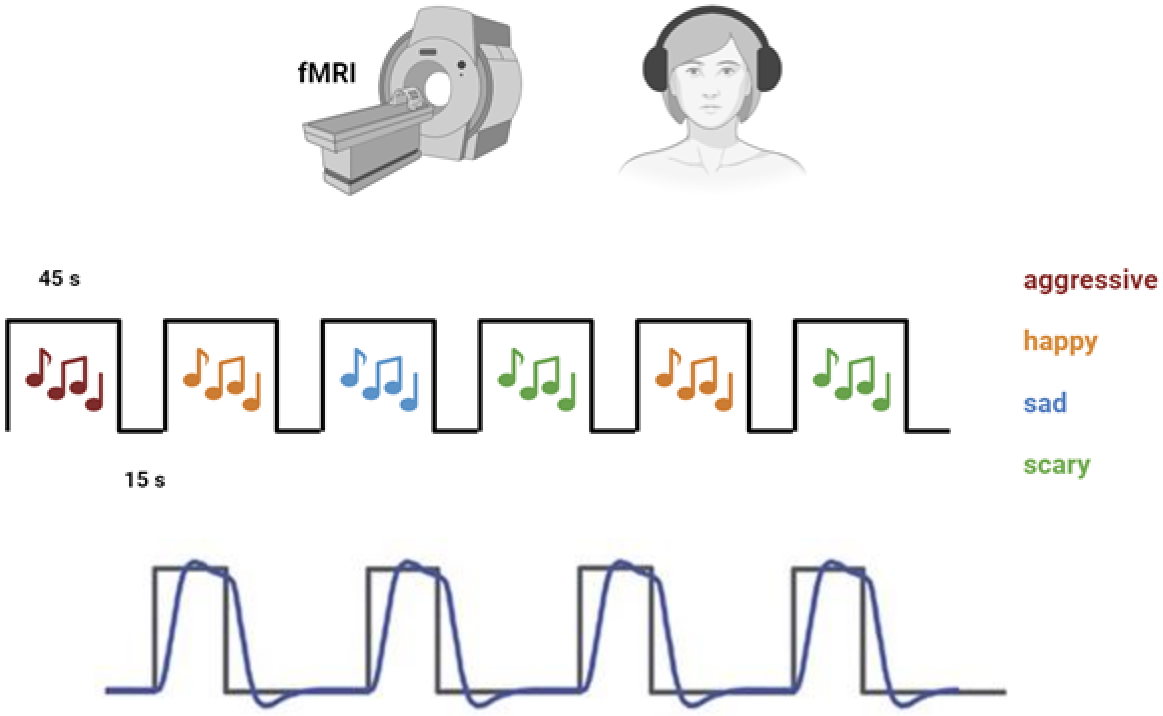
# Limitations

- The model is the same for each voxel (mass univariate), but the haemodynamic response might differ across the brain regions.

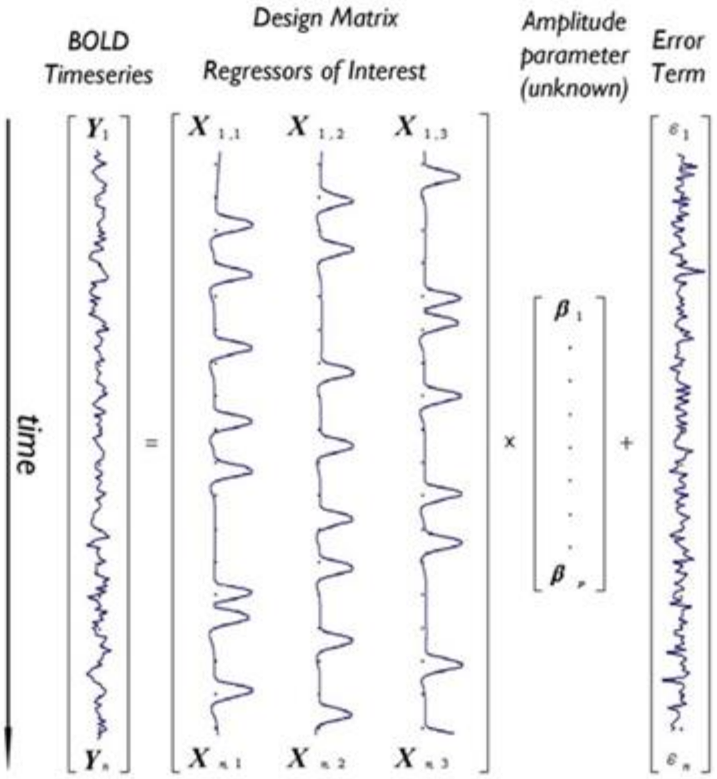
- The model assumes that noise varies with a normal distribution in each voxel in each time point, but e.g. noise differs greatly close to large arteries.

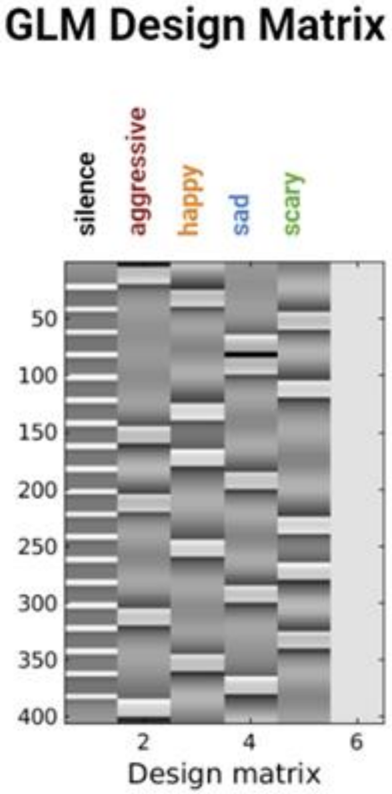
- The model assumes independent statistical test for each voxel, but voxels in the same vicinity have similar properties.

# Case Study



# Case Study


BOLD Timeseries
Design Matrix
Regressors of Interest
Amplitude parameter (unknown)
Error Term
time
=
×
+


GLM Design Matrix
silence
aggressive
happy
sad
scary
50
100
150
200
250
300
350
400
2
4
6
Design matrix

# Quiz 1

How do you get the predicted response model ?

By convolving the known stimuli design with the haemodynamic response function

# Quiz 2

How does one fit the best GLModel possible ?

1) Minimising the residuals
2) Including relevant confounds in GLM
3) Including relevant covariates

# Quiz 3

Will including an additional regressor in the GLM affect the parameter estimates ?

Yes

# Take Home Messages

fMRI GLM analysis

- ➔ determines where there is a significant relationship between the measured BOLD signal and the predicted response

- ➔ provides a set of whole brain maps of beta values (one for each regressor)

# Thank you for your attention !

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